Computer algebra independent integration tests

5-Inverse-trig-functions/5.3-Inverse-tangent/5.3.3-d+e-x-^m-a+b-arctan-c-x^n-^p

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [31]. This is test number [149].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 93.55 (29)	% 6.45 (2)
Mathematica	% 83.87 (26)	% 16.13 (5)
Maple	% 96.77 (30)	% 3.23 (1)
Maxima	% 45.16 (14)	% 54.84 (17)
Fricas	% 35.48 (11)	% 64.52 (20)
Sympy	% 29.03 (9)	% 70.97 (22)
Giac	% 45.16 (14)	% 54.84 (17)

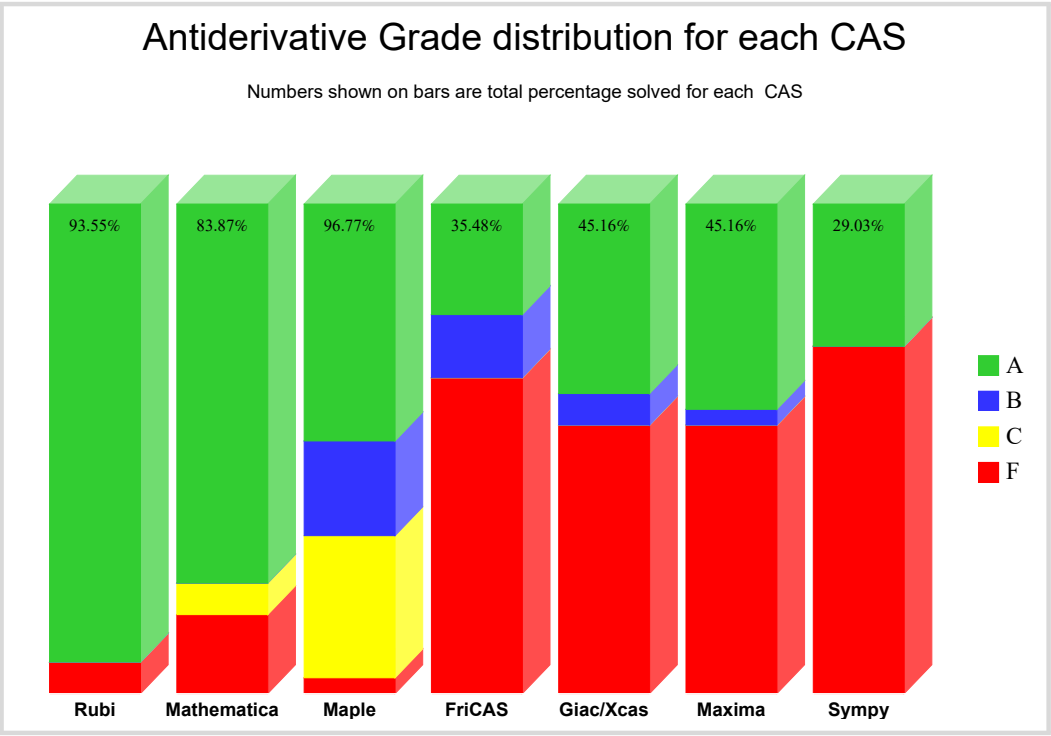
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

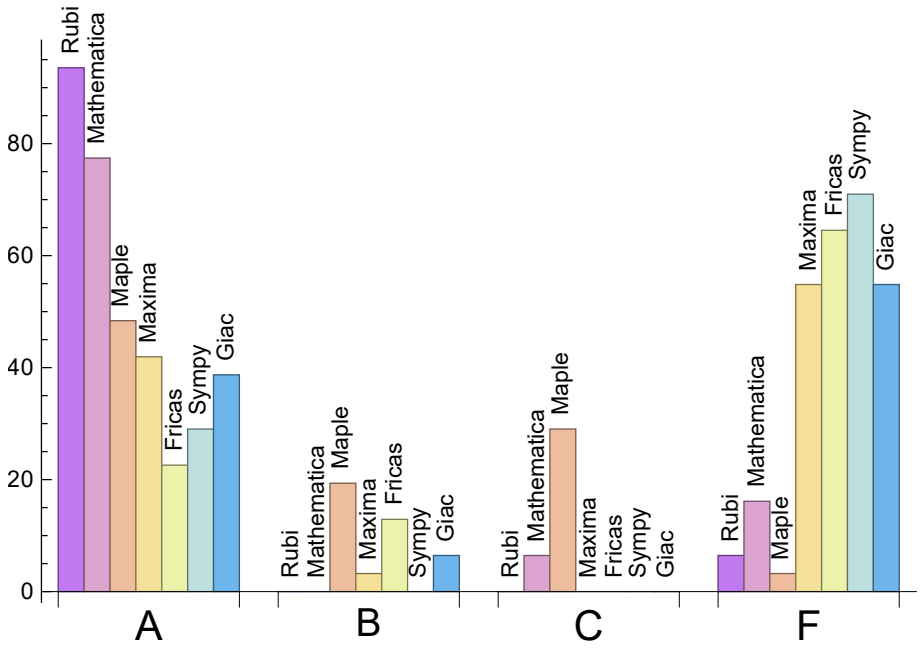
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	93.55	0.	0.	6.45
Mathematica	77.42	0.	6.45	16.13
Maple	48.39	19.35	29.03	3.23
Maxima	41.94	3.23	0.	54.84
Fricas	22.58	12.9	0.	64.52
Sympy	29.03	0.	0.	70.97
Giac	38.71	6.45	0.	54.84

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.52	341.34	0.94	264.	1.
Mathematica	14.53	490.35	1.15	276.	1.01
Maple	0.63	2349.07	4.5	370.5	1.48
Maxima	1.39	390.57	1.68	361.5	1.7
Fricas	2.83	1337.73	6.35	410.	2.85
Sympy	34.45	726.22	3.96	262.	1.82
Giac	3.09	350.86	1.89	335.5	1.82

1.4 list of integrals that has no closed form antiderivative

{26, 27}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {25}

Mathematica {5, 9, 10, 11, 13, 14, 15, 16, 17, 25}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

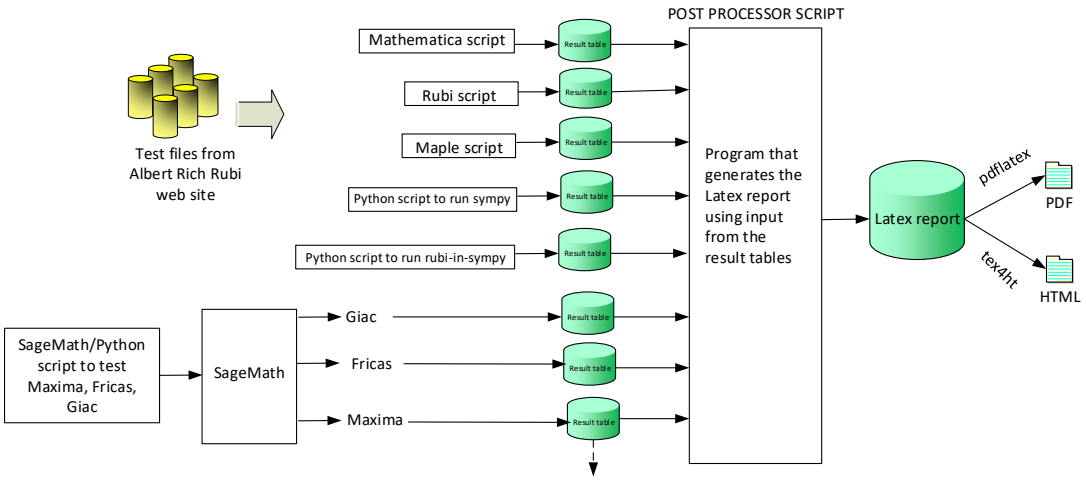
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 31 }

B grade: { }

C grade: { }

F grade: { 23, 30}

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 21, 22, 24, 26, 27, 28, 29, 31 }

B grade: { }

C grade: { 23, 25 }

F grade: { 12, 18, 19, 20, 30}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 21, 22, 24, 26, 27, 29, 31 }

B grade: { 9, 10, 11, 13, 14, 28 }

C grade: { 12, 15, 16, 17, 18, 19, 20, 23, 30 }

F grade: { 25 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 6, 7, 8, 22, 24, 26, 28, 29, 31

B grade: { 21 }

C grade: { }

F grade: { 5, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 23, 25, 27, 30 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 6, 26, 27 }

B grade: { 7, 8, 21, 22 }

C grade: { }

F grade: { 5, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 23, 24, 25, 28, 29, 30, 31 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 6, 21, 22, 28, 29 }

B grade: { }

C grade: { }

F grade: { 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 23, 24, 25, 26, 27, 30, 31 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 6, 21, 22, 24, 26, 27, 28, 29 }

B grade: { 7, 8 }

C grade: { }

F grade: { 5, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 23, 25, 30, 31 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as

F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	255	283	340	564	345	425
normalized size	1	1.	1.39	1.54	1.85	3.07	1.88	2.31
time (sec)	N/A	0.143	0.476	0.027	1.505	2.633	3.329	1.249

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	218	207	251	410	262	316
normalized size	1	1.	1.51	1.44	1.74	2.85	1.82	2.19
time (sec)	N/A	0.123	0.452	0.028	1.465	2.33	2.219	1.203

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	163	137	170	281	160	217
normalized size	1	1.	1.58	1.33	1.65	2.73	1.55	2.11
time (sec)	N/A	0.09	0.329	0.027	1.466	2.301	1.31	1.139

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	77	68	96	162	87	122
normalized size	1	1.	1.01	0.89	1.26	2.13	1.14	1.61
time (sec)	N/A	0.061	0.005	0.024	1.486	2.116	0.731	1.222

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	138	138	138	168	0	0	0	0
normalized size	1	1.	1.	1.22	0.	0.	0.	0.
time (sec)	N/A	0.083	0.06	0.054	0.	0.	0.	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	111	118	144	259	777	217
normalized size	1	1.	1.13	1.2	1.47	2.64	7.93	2.21
time (sec)	N/A	0.053	0.195	0.031	1.481	2.505	46.256	1.146

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	192	184	289	637	0	489
normalized size	1	1.	1.32	1.26	1.98	4.36	0.	3.35
time (sec)	N/A	0.123	0.327	0.034	1.47	3.64	0.	1.634

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	254	282	505	1280	0	1042
normalized size	1	1.	1.23	1.37	2.45	6.21	0.	5.06
time (sec)	N/A	0.182	0.641	0.036	1.556	8.105	0.	10.9

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	376	376	472	948	0	0	0	0
normalized size	1	1.	1.26	2.52	0.	0.	0.	0.
time (sec)	N/A	0.573	0.946	0.075	0.	0.	0.	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	270	270	312	750	0	0	0	0
normalized size	1	1.	1.16	2.78	0.	0.	0.	0.
time (sec)	N/A	0.399	0.573	0.069	0.	0.	0.	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	171	171	172	360	0	0	0	0
normalized size	1	1.	1.01	2.11	0.	0.	0.	0.
time (sec)	N/A	0.298	0.269	0.091	0.	0.	0.	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	223	223	0	1297	0	0	0	0
normalized size	1	1.	0.	5.82	0.	0.	0.	0.
time (sec)	N/A	0.049	114.81	0.911	0.	0.	0.	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	341	341	300	698	0	0	0	0
normalized size	1	1.	0.88	2.05	0.	0.	0.	0.
time (sec)	N/A	0.37	2.881	0.102	0.	0.	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	496	496	479	961	0	0	0	0
normalized size	1	1.	0.97	1.94	0.	0.	0.	0.
time (sec)	N/A	0.539	6.038	0.102	0.	0.	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	652	652	855	3577	0	0	0	0
normalized size	1	1.	1.31	5.49	0.	0.	0.	0.
time (sec)	N/A	1.202	1.842	3.813	0.	0.	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	411	411	621	3022	0	0	0	0
normalized size	1	1.	1.51	7.35	0.	0.	0.	0.
time (sec)	N/A	0.774	1.117	2.606	0.	0.	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	264	264	342	7462	0	0	0	0
normalized size	1	1.	1.3	28.27	0.	0.	0.	0.
time (sec)	N/A	0.58	0.594	0.813	0.	0.	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	320	320	0	2616	0	0	0	0
normalized size	1	1.	0.	8.18	0.	0.	0.	0.
time (sec)	N/A	0.057	180.004	0.573	0.	0.	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	499	499	0	2960	0	0	0	0
normalized size	1	1.	0.	5.93	0.	0.	0.	0.
time (sec)	N/A	0.53	123.733	0.774	0.	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F(-1)	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	936	936	0	41013	0	0	0	0
normalized size	1	1.	0.	43.82	0.	0.	0.	0.
time (sec)	N/A	1.09	67.124	6.972	0.	0.	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	252	381	771	9956	3135	510
normalized size	1	1.	1.01	1.52	3.08	39.82	12.54	2.04
time (sec)	N/A	0.303	3.241	0.033	1.54	4.646	49.978	1.881

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	191	153	167	404	1166	1515	271
normalized size	1	0.99	0.8	0.87	2.1	6.07	7.89	1.41
time (sec)	N/A	0.209	0.095	0.028	1.501	2.811	31.707	1.263

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	C	F	F	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	501	0	326	138	0	0	0	0
normalized size	1	0.	0.65	0.28	0.	0.	0.	0.
time (sec)	N/A	0.064	32.123	0.125	0.	0.	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	328	321	433	649	0	0	512
normalized size	1	1.	0.98	1.32	1.98	0.	0.	1.56
time (sec)	N/A	0.522	0.745	0.036	1.485	0.	0.	14.297

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	1325	1554	5593	0	0	0	0	0
normalized size	1	1.17	4.22	0.	0.	0.	0.	0.
time (sec)	N/A	3.099	32.199	0.277	0.	0.	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.123	70.59	0.543	0.	0.	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.656	63.716	0.49	0.	0.	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	331	297	536	448	0	151	436
normalized size	1	1.05	0.94	1.7	1.42	0.	0.48	1.38
time (sec)	N/A	0.709	143.84	0.095	1.459	0.	113.241	4.898

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	310	314	383	0	104	355
normalized size	1	1.	1.09	1.1	1.34	0.	0.36	1.25
time (sec)	N/A	0.603	0.089	0.069	1.473	0.	61.318	2.48

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	C	F	F	F(-1)	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	739	0	0	172	0	0	0	0
normalized size	1	0.	0.	0.23	0.	0.	0.	0.
time (sec)	N/A	0.063	180.003	0.128	0.	0.	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	906	906	536	1220	1018	0	0	0
normalized size	1	1.	0.59	1.35	1.12	0.	0.	0.
time (sec)	N/A	1.484	14.463	0.139	1.57	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [25] had the largest ratio of [2.556]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	5	1.	16	0.312
2	A	6	5	1.	16	0.312
3	A	6	5	1.	16	0.312
4	A	6	5	1.	14	0.357
5	A	4	4	1.	16	0.25
6	A	6	6	1.	16	0.375
7	A	7	6	1.	16	0.375
8	A	7	6	1.	16	0.375

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
9	A	19	14	1.	18	0.778
10	A	15	12	1.	18	0.667
11	A	12	9	1.	16	0.562
12	A	1	1	1.	18	0.056
13	A	13	9	1.	18	0.5
14	A	19	15	1.	18	0.833
15	A	29	15	1.	18	0.833
16	A	20	13	1.	18	0.722
17	A	14	10	1.	16	0.625
18	A	1	1	1.	18	0.056
19	A	10	8	1.	18	0.444
20	A	23	12	1.	18	0.667
21	A	18	14	1.	18	0.778
22	A	16	10	0.99	16	0.625
23	F	0	0	N/A	0	N/A
24	A	19	14	1.	18	0.778
25	A	110	46	1.17	18	2.556
26	A	0	0	0.	0	0.
27	A	0	0	0.	0	0.
28	A	25	14	1.05	18	0.778
29	A	23	13	1.	16	0.812
30	F	0	0	N/A	0	N/A
31	A	35	16	1.	18	0.889

Chapter 3

Listing of integrals

3.1 $\int (d+ex)^4 \left(a+b\tan^{-1}(cx)\right)\,dx$

Optimal. Leaf size=184

$$\frac{(d+ex)^5\left(a+b\tan^{-1}(cx)\right)}{5e}-\frac{be^2x^2\left(10c^2d^2-e^2\right)}{10c^3}-\frac{b\left(-10c^2d^2e^2+5c^4d^4+e^4\right)\log\left(c^2x^2+1\right)}{10c^5}-\frac{bdex\left(2c^2d^2-e^2\right)}{c^3}-\frac{bd}{}$$

```
[Out] -((b*d*e*(2*c^2*d^2 - e^2)*x)/c^3) - (b*e^2*(10*c^2*d^2 - e^2)*x^2)/(10*c^3
) - (b*d*e^3*x^3)/(3*c) - (b*e^4*x^4)/(20*c) - (b*d*(c^4*d^4 - 10*c^2*d^2*e
^2 + 5*e^4)*ArcTan[c*x])/(5*c^4*e) + ((d + e*x)^5*(a + b*ArcTan[c*x]))/(5*e
) - (b*(5*c^4*d^4 - 10*c^2*d^2*e^2 + e^4)*Log[1 + c^2*x^2])/(10*c^5)
```

Rubi [A] time = 0.142807, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {4862, 702, 635, 203, 260}

$$\frac{(d+ex)^5\left(a+b\tan^{-1}(cx)\right)}{5e}-\frac{be^2x^2\left(10c^2d^2-e^2\right)}{10c^3}-\frac{b\left(-10c^2d^2e^2+5c^4d^4+e^4\right)\log\left(c^2x^2+1\right)}{10c^5}-\frac{bdex\left(2c^2d^2-e^2\right)}{c^3}-\frac{bd}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4*(a + b*ArcTan[c*x]),x]
```

```
[Out] -((b*d*e*(2*c^2*d^2 - e^2)*x)/c^3) - (b*e^2*(10*c^2*d^2 - e^2)*x^2)/(10*c^3
) - (b*d*e^3*x^3)/(3*c) - (b*e^4*x^4)/(20*c) - (b*d*(c^4*d^4 - 10*c^2*d^2*e
^2 + 5*e^4)*ArcTan[c*x])/(5*c^4*e) + ((d + e*x)^5*(a + b*ArcTan[c*x]))/(5*e
```

```
) - (b*(5*c^4*d^4 - 10*c^2*d^2*e^2 + e^4)*Log[1 + c^2*x^2])/(10*c^5)
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int (d+ex)^4\left(a+b\tan^{-1}(cx)\right)dx = \frac{(d+ex)^5\left(a+b\tan^{-1}(cx)\right)}{5e} - \frac{(bc)\int \frac{(d+ex)^5}{1+c^2x^2}\,dx}{5e}$$

$$= \frac{(d+ex)^5\left(a+b\tan^{-1}(cx)\right)}{5e} - \frac{(bc)\int \left(\frac{5de^2\left(2c^2d^2-e^2\right)}{c^4} + \frac{e^3\left(10c^2d^2-e^2\right)x}{c^4} + \frac{5de^4x^2}{c^2} + \frac{e^5x^3}{c^2} + \frac{c^4}{}\right.}{5e}$$

$$= -\frac{bde\left(2c^2d^2-e^2\right)x}{c^3} - \frac{be^2\left(10c^2d^2-e^2\right)x^2}{10c^3} - \frac{bde^3x^3}{3c} - \frac{be^4x^4}{20c} + \frac{(d+ex)^5\left(a+b\tan^{-1}\right.}{5e}$$

$$= -\frac{bde\left(2c^2d^2-e^2\right)x}{c^3} - \frac{be^2\left(10c^2d^2-e^2\right)x^2}{10c^3} - \frac{bde^3x^3}{3c} - \frac{be^4x^4}{20c} + \frac{(d+ex)^5\left(a+b\tan^{-1}\right.}{5e}$$

$$= -\frac{bde\left(2c^2d^2-e^2\right)x}{c^3} - \frac{be^2\left(10c^2d^2-e^2\right)x^2}{10c^3} - \frac{bde^3x^3}{3c} - \frac{be^4x^4}{20c} - \frac{bd\left(c^4d^4-10c^2d^2e^2-\right.}{5c^4e}$$

Mathematica [A] time = 0.475795, size = 255, normalized size = 1.39

$$\frac{(d+ex)^5\left(a+b\tan^{-1}(cx)\right) - \frac{b\left(c^2e^2x\left(c^2\left(60d^2ex+120d^3+20de^2x^2+3e^3x^3\right)-6e^2(10d+ex)\right)+6\left(-10c^2d^2e^2\left(\sqrt{-c^2}d+e\right)+c^4d^4\left(\sqrt{-c^2}d+5e\right)+e^4\left(5\sqrt{-c^2}d+e\right)\right.\right.}{12c^5}}{5e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4*(a + b*ArcTan[c*x]),x]
```

```
[Out] ((d + e*x)^5*(a + b*ArcTan[c*x]) - (b*(c^2*e^2*x*(-6*e^2*(10*d + e*x) + c^2
*(120*d^3 + 60*d^2*e*x + 20*d*e^2*x^2 + 3*e^3*x^3)) + 6*(-10*c^2*d^2*e^2*(S
qrt[-c^2]*d + e) + e^4*(5*Sqrt[-c^2]*d + e) + c^4*d^4*(Sqrt[-c^2]*d + 5*e))
*Log[1 - Sqrt[-c^2]*x] - 6*(c^4*d^4*(Sqrt[-c^2]*d - 5*e) - 10*c^2*d^2*(Sqrt
[-c^2]*d - e)*e^2 + (5*Sqrt[-c^2]*d - e)*e^4)*Log[1 + Sqrt[-c^2]*x]))/(12*c
^5))/(5*e)
```

Maple [A] time = 0.027, size = 283, normalized size = 1.5

$$\frac{ae^4x^5}{5} + ae^3x^4d + 2\,ae^2x^3d^2 + 2\,aex^2d^3 + axd^4 + \frac{ad^5}{5e} + \frac{be^4\arctan(cx)\,x^5}{5} + be^3\arctan(cx)\,x^4d + 2\,be^2\arctan(cx)\,x^3d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4*(a+b*arctan(c*x)),x)
```

```
[Out] 1/5*a*e^4*x^5+a*e^3*x^4*d+2*a*e^2*x^3*d^2+2*a*e*x^2*d^3+a*x*d^4+1/5*a/e*d^5
+1/5*b*e^4*arctan(c*x)*x^5+b*e^3*arctan(c*x)*x^4*d+2*b*e^2*arctan(c*x)*x^3*
d^2+2*b*e*arctan(c*x)*x^2*d^3+b*arctan(c*x)*x*d^4-1/20*b*e^4*x^4/c-1/3*b*d*
e^3*x^3/c-1/c*b*e^2*x^2*d^2-2*b/c*e*d^3*x+1/10/c^3*b*e^4*x^2+b/c^3*e^3*d*x-
1/2/c*b*ln(c^2*x^2+1)*d^4+1/c^3*b*e^2*ln(c^2*x^2+1)*d^2-1/10/c^5*b*e^4*ln(c
^2*x^2+1)+2/c^2*b*e*arctan(c*x)*d^3-1/c^4*b*e^3*arctan(c*x)*d
```

Maxima [A] time = 1.50502, size = 340, normalized size = 1.85

$$\frac{1}{5}ae^4x^5 + ade^3x^4 + 2ad^2e^2x^3 + 2ad^3ex^2 + 2\left(x^2\arctan(cx) - c\left(\frac{x}{c^2} - \frac{\arctan(cx)}{c^3}\right)\right)bd^3e + \left(2x^3\arctan(cx) - c\left(\frac{x^2}{c^2} - \frac{\text{lo}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*e^4*x^5 + a*d*e^3*x^4 + 2*a*d^2*e^2*x^3 + 2*a*d^3*e*x^2 + 2*(x^2*arct
an(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b*d^3*e + (2*x^3*arctan(c*x) - c*(x^
2/c^2 - log(c^2*x^2 + 1)/c^4))*b*d^2*e^2 + 1/3*(3*x^4*arctan(c*x) - c*((c^2
*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*d*e^3 + 1/20*(4*x^5*arctan(c*x) - c
*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*e^4 + a*d^4*x + 1/2*(2
*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d^4/c
```

Fricas [A] time = 2.63284, size = 564, normalized size = 3.07

$$\frac{12ac^5e^4x^5 + 3\left(20ac^5de^3 - bc^4e^4\right)x^4 + 20\left(6ac^5d^2e^2 - bc^4de^3\right)x^3 + 6\left(20ac^5d^3e - 10bc^4d^2e^2 + bc^2e^4\right)x^2 + 60\left(ac^5d^4 - 2b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/60*(12*a*c^5*e^4*x^5 + 3*(20*a*c^5*d*e^3 - b*c^4*e^4)*x^4 + 20*(6*a*c^5*d
^2*e^2 - b*c^4*d*e^3)*x^3 + 6*(20*a*c^5*d^3*e - 10*b*c^4*d^2*e^2 + b*c^2*e^
4)*x^2 + 60*(a*c^5*d^4 - 2*b*c^4*d^3*e + b*c^2*d*e^3)*x + 12*(b*c^5*e^4*x^5
 + 5*b*c^5*d*e^3*x^4 + 10*b*c^5*d^2*e^2*x^3 + 10*b*c^5*d^3*e*x^2 + 5*b*c^5*
d^4*x + 10*b*c^3*d^3*e - 5*b*c*d*e^3)*arctan(c*x) - 6*(5*b*c^4*d^4 - 10*b*c
^2*d^2*e^2 + b*e^4)*log(c^2*x^2 + 1))/c^5
```

Sympy [A] time = 3.3293, size = 345, normalized size = 1.88

$$\begin{cases} ad^4x + 2ad^3ex^2 + 2ad^2e^2x^3 + ade^3x^4 + \frac{ae^4x^5}{5} + bd^4x\operatorname{atan}(cx) + 2bd^3ex^2\operatorname{atan}(cx) + 2bd^2e^2x^3\operatorname{atan}(cx) + bde^3x^4\operatorname{atan} \\ a\left(d^4x + 2d^3ex^2 + 2d^2e^2x^3 + de^3x^4 + \frac{e^4x^5}{5}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*d**4*x + 2*a*d**3*e*x**2 + 2*a*d**2*e**2*x**3 + a*d*e**3*x**4
+ a*e**4*x**5/5 + b*d**4*x*atan(c*x) + 2*b*d**3*e*x**2*atan(c*x) + 2*b*d**2
*e**2*x**3*atan(c*x) + b*d*e**3*x**4*atan(c*x) + b*e**4*x**5*atan(c*x)/5 -
b*d**4*log(x**2 + c**(-2))/(2*c) - 2*b*d**3*e*x/c - b*d**2*e**2*x**2/c - b*
d*e**3*x**3/(3*c) - b*e**4*x**4/(20*c) + 2*b*d**3*e*atan(c*x)/c**2 + b*d**2
*e**2*log(x**2 + c**(-2))/c**3 + b*d*e**3*x/c**3 + b*e**4*x**2/(10*c**3) -
b*d*e**3*atan(c*x)/c**4 - b*e**4*log(x**2 + c**(-2))/(10*c**5), Ne(c, 0)),
(a*(d**4*x + 2*d**3*e*x**2 + 2*d**2*e**2*x**3 + d*e**3*x**4 + e**4*x**5/5),
 True))
```

Giac [A] time = 1.24916, size = 425, normalized size = 2.31

$$\frac{12\,bc^5x^5\arctan(cx)\,e^4 + 60\,bc^5dx^4\arctan(cx)\,e^3 + 120\,bc^5d^2x^3\arctan(cx)\,e^2 + 120\,bc^5d^3x^2\arctan(cx)\,e + 60\,bc^5d^4x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] 1/60*(12*b*c^5*x^5*arctan(c*x)*e^4 + 60*b*c^5*d*x^4*arctan(c*x)*e^3 + 120*b
*c^5*d^2*x^3*arctan(c*x)*e^2 + 120*b*c^5*d^3*x^2*arctan(c*x)*e + 60*b*c^5*d
^4*x*arctan(c*x) + 12*a*c^5*x^5*e^4 + 60*a*c^5*d*x^4*e^3 + 120*a*c^5*d^2*x^
3*e^2 + 120*a*c^5*d^3*x^2*e + 60*a*c^5*d^4*x - 120*pi*b*c^3*d^3*e*sgn(c)*sg
n(x) - 3*b*c^4*x^4*e^4 - 20*b*c^4*d*x^3*e^3 - 60*b*c^4*d^2*x^2*e^2 - 120*b*
c^4*d^3*x*e - 30*b*c^4*d^4*log(c^2*x^2 + 1) + 120*b*c^3*d^3*arctan(c*x)*e +
 60*b*c^2*d^2*e^2*log(c^2*x^2 + 1) + 6*b*c^2*x^2*e^4 + 60*b*c^2*d*x*e^3 - 6
0*b*c*d*arctan(c*x)*e^3 - 6*b*e^4*log(c^2*x^2 + 1))/c^5
```

3.2 $\int (d+ex)^3\left(a+b\tan^{-1}(cx)\right)\,dx$

Optimal. Leaf size=144

$$\frac{(d+ex)^4\left(a+b\tan^{-1}(cx)\right)}{4e}-\frac{bex\left(6c^2d^2-e^2\right)}{4c^3}-\frac{b\left(-6c^2d^2e^2+c^4d^4+e^4\right)\tan^{-1}(cx)}{4c^4e}-\frac{bd(cd-e)(cd+e)\log\left(c^2x^2+1\right)}{2c^3}-$$

```
[Out] -(b*e*(6*c^2*d^2 - e^2)*x)/(4*c^3) - (b*d*e^2*x^2)/(2*c) - (b*e^3*x^3)/(12*
c) - (b*(c^4*d^4 - 6*c^2*d^2*e^2 + e^4)*ArcTan[c*x])/(4*c^4*e) + ((d + e*x)
^4*(a + b*ArcTan[c*x]))/(4*e) - (b*d*(c*d - e)*(c*d + e)*Log[1 + c^2*x^2])/
(2*c^3)
```

Rubi [A] time = 0.122858, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {4862, 702, 635, 203, 260}

$$\frac{(d+ex)^4\left(a+b\tan^{-1}(cx)\right)}{4e}-\frac{bex\left(6c^2d^2-e^2\right)}{4c^3}-\frac{b\left(-6c^2d^2e^2+c^4d^4+e^4\right)\tan^{-1}(cx)}{4c^4e}-\frac{bd(cd-e)(cd+e)\log\left(c^2x^2+1\right)}{2c^3}-$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*ArcTan[c*x]),x]
```

```
[Out] -(b*e*(6*c^2*d^2 - e^2)*x)/(4*c^3) - (b*d*e^2*x^2)/(2*c) - (b*e^3*x^3)/(12*
c) - (b*(c^4*d^4 - 6*c^2*d^2*e^2 + e^4)*ArcTan[c*x])/(4*c^4*e) + ((d + e*x)
^4*(a + b*ArcTan[c*x]))/(4*e) - (b*d*(c*d - e)*(c*d + e)*Log[1 + c^2*x^2])/
(2*c^3)
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int (d+ex)^3\left(a+b\tan^{-1}(cx)\right)dx = \frac{(d+ex)^4\left(a+b\tan^{-1}(cx)\right)}{4e} - \frac{(bc)\int \frac{(d+ex)^4}{1+c^2x^2}\,dx}{4e}$$

$$= \frac{(d+ex)^4\left(a+b\tan^{-1}(cx)\right)}{4e} - \frac{(bc)\int \left(\frac{e^2\left(6c^2d^2-e^2\right)}{c^4} + \frac{4de^3x}{c^2} + \frac{e^4x^2}{c^2} + \frac{c^4d^4-6c^2d^2e^2+e^4+4c^2d(c}{c^4\left(1+c^2x^2\right)}\right.}{4e}$$

$$= -\frac{be\left(6c^2d^2-e^2\right)x}{4c^3} - \frac{bde^2x^2}{2c} - \frac{be^3x^3}{12c} + \frac{(d+ex)^4\left(a+b\tan^{-1}(cx)\right)}{4e} - \frac{b\int \frac{c^4d^4-6c^2d^2e^2}{}}{}$$

$$= -\frac{be\left(6c^2d^2-e^2\right)x}{4c^3} - \frac{bde^2x^2}{2c} - \frac{be^3x^3}{12c} + \frac{(d+ex)^4\left(a+b\tan^{-1}(cx)\right)}{4e} - \frac{(bd(cd-e)(cd}{}$$

$$= -\frac{be\left(6c^2d^2-e^2\right)x}{4c^3} - \frac{bde^2x^2}{2c} - \frac{be^3x^3}{12c} - \frac{b\left(c^4d^4-6c^2d^2e^2+e^4\right)\tan^{-1}(cx)}{4c^4e} + \frac{(d+ex)}{}$$

Mathematica [A] time = 0.452377, size = 218, normalized size = 1.51

$$\frac{(d+ex)^4\left(a+b\tan^{-1}(cx)\right) - \frac{bc\left(2\sqrt{-c^2}e^2x\left(c^2\left(18d^2+6dex+e^2x^2\right)-3e^2\right)-3\left(-2c^2d^2e\left(2\sqrt{-c^2}d+3e\right)+c^4d^4+e^3\left(4\sqrt{-c^2}d+e\right)\right)\log\left(1-\sqrt{-c^2}x\right)+3\left(2c^2d^2e\left(2\sqrt{-c^2}\right.\right.\right.}{6\left(-c^2\right)^{5/2}}}{4e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*ArcTan[c*x]),x]
```

```
[Out] ((d + e*x)^4*(a + b*ArcTan[c*x]) - (b*c*(2*Sqrt[-c^2]*e^2*x*(-3*e^2 + c^2*(
18*d^2 + 6*d*e*x + e^2*x^2)) - 3*(c^4*d^4 + e^3*(4*Sqrt[-c^2]*d + e) - 2*c^
2*d^2*e*(2*Sqrt[-c^2]*d + 3*e))*Log[1 - Sqrt[-c^2]*x] + 3*(c^4*d^4 + 2*c^2*
d^2*(2*Sqrt[-c^2]*d - 3*e)*e + e^3*(-4*Sqrt[-c^2]*d + e))*Log[1 + Sqrt[-c^2
]*x]))/(6*(-c^2)^(5/2)))/(4*e)
```

Maple [A] time = 0.028, size = 207, normalized size = 1.4

$$\frac{ae^3x^4}{4} + ae^2x^3d + \frac{3\,aex^2d^2}{2} + axd^3 + \frac{ad^4}{4\,e} + \frac{be^3\arctan(cx)\,x^4}{4} + be^2\arctan(cx)\,x^3d + \frac{3\,be\arctan(cx)\,x^2d^2}{2} + b\arctan($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(a+b*arctan(c*x)),x)
```

```
[Out] 1/4*a*e^3*x^4+a*e^2*x^3*d+3/2*a*e*x^2*d^2+a*x*d^3+1/4*a/e*d^4+1/4*b*e^3*arc
tan(c*x)*x^4+b*e^2*arctan(c*x)*x^3*d+3/2*b*e*arctan(c*x)*x^2*d^2+b*arctan(c
*x)*x*d^3-1/12*b*e^3*x^3/c-1/2*b*d*e^2*x^2/c-3/2*b/c*e*d^2*x+1/4*b/c^3*e^3*
x-1/2/c*b*ln(c^2*x^2+1)*d^3+1/2/c^3*b*e^2*ln(c^2*x^2+1)*d+3/2/c^2*b*e*arcta
n(c*x)*d^2-1/4/c^4*b*e^3*arctan(c*x)
```

Maxima [A] time = 1.46532, size = 251, normalized size = 1.74

$$\frac{1}{4}\,ae^3x^4 + ade^2x^3 + \frac{3}{2}\,ad^2ex^2 + \frac{3}{2}\left(x^2\arctan(cx) - c\left(\frac{x}{c^2} - \frac{\arctan(cx)}{c^3}\right)\right)bd^2e + \frac{1}{2}\left(2\,x^3\arctan(cx) - c\left(\frac{x^2}{c^2} - \frac{\log\left(c^2x^2 +\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*e^3*x^4 + a*d*e^2*x^3 + 3/2*a*d^2*e*x^2 + 3/2*(x^2*arctan(c*x) - c*(x
/c^2 - arctan(c*x)/c^3))*b*d^2*e + 1/2*(2*x^3*arctan(c*x) - c*(x^2/c^2 - lo
g(c^2*x^2 + 1)/c^4))*b*d*e^2 + 1/12*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)
/c^4 + 3*arctan(c*x)/c^5))*b*e^3 + a*d^3*x + 1/2*(2*c*x*arctan(c*x) - log(c
^2*x^2 + 1))*b*d^3/c
```

Fricas [A] time = 2.32974, size = 410, normalized size = 2.85

$$\frac{3\,ac^4e^3x^4 + \left(12\,ac^4de^2 - bc^3e^3\right)x^3 + 6\left(3\,ac^4d^2e - bc^3de^2\right)x^2 + 3\left(4\,ac^4d^3 - 6\,bc^3d^2e + bce^3\right)x + 3\left(bc^4e^3x^4 + 4\,bc^4de^2x^3 +}{12\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*c^4*e^3*x^4 + (12*a*c^4*d*e^2 - b*c^3*e^3)*x^3 + 6*(3*a*c^4*d^2*e
 - b*c^3*d*e^2)*x^2 + 3*(4*a*c^4*d^3 - 6*b*c^3*d^2*e + b*c*e^3)*x + 3*(b*c^
4*e^3*x^4 + 4*b*c^4*d*e^2*x^3 + 6*b*c^4*d^2*e*x^2 + 4*b*c^4*d^3*x + 6*b*c^2
*d^2*e - b*e^3)*arctan(c*x) - 6*(b*c^3*d^3 - b*c*d*e^2)*log(c^2*x^2 + 1))/c
^4
```

Sympy [A] time = 2.21886, size = 262, normalized size = 1.82

$$\begin{cases} ad^3x + \frac{3ad^2ex^2}{2} + ade^2x^3 + \frac{ae^3x^4}{4} + bd^3x\operatorname{atan}(cx) + \frac{3bd^2ex^2\operatorname{atan}(cx)}{2} + bde^2x^3\operatorname{atan}(cx) + \frac{be^3x^4\operatorname{atan}(cx)}{4} - \frac{bd^3\log\left(x^2+\frac{1}{c^2}\right)}{2c} - \frac{3} \\ a\left(d^3x + \frac{3d^2ex^2}{2} + de^2x^3 + \frac{e^3x^4}{4}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*d**3*x + 3*a*d**2*e*x**2/2 + a*d*e**2*x**3 + a*e**3*x**4/4 + b
*d**3*x*atan(c*x) + 3*b*d**2*e*x**2*atan(c*x)/2 + b*d*e**2*x**3*atan(c*x) +
 b*e**3*x**4*atan(c*x)/4 - b*d**3*log(x**2 + c**(-2))/(2*c) - 3*b*d**2*e*x/
(2*c) - b*d*e**2*x**2/(2*c) - b*e**3*x**3/(12*c) + 3*b*d**2*e*atan(c*x)/(2*
c**2) + b*d*e**2*log(x**2 + c**(-2))/(2*c**3) + b*e**3*x/(4*c**3) - b*e**3*
atan(c*x)/(4*c**4), Ne(c, 0)), (a*(d**3*x + 3*d**2*e*x**2/2 + d*e**2*x**3 +
 e**3*x**4/4), True))
```

Giac [A] time = 1.2029, size = 316, normalized size = 2.19

$$\frac{3\,bc^4x^4\arctan(cx)\,e^3 + 12\,bc^4dx^3\arctan(cx)\,e^2 + 18\,bc^4d^2x^2\arctan(cx)\,e + 12\,bc^4d^3x\arctan(cx) + 3\,ac^4x^4e^3 + 12\,ac^}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] 1/12*(3*b*c^4*x^4*arctan(c*x)*e^3 + 12*b*c^4*d*x^3*arctan(c*x)*e^2 + 18*b*c
^4*d^2*x^2*arctan(c*x)*e + 12*b*c^4*d^3*x*arctan(c*x) + 3*a*c^4*x^4*e^3 + 1
2*a*c^4*d*x^3*e^2 + 18*a*c^4*d^2*x^2*e + 12*a*c^4*d^3*x - 18*pi*b*c^2*d^2*e
*sgn(c)*sgn(x) - b*c^3*x^3*e^3 - 6*b*c^3*d*x^2*e^2 - 18*b*c^3*d^2*x*e - 6*b
*c^3*d^3*log(c^2*x^2 + 1) + 18*b*c^2*d^2*arctan(c*x)*e + 6*b*c*d*e^2*log(c^
2*x^2 + 1) + 3*b*c*x*e^3 - 3*b*arctan(c*x)*e^3)/c^4
```

3.3 $\int (d+ex)^2\left(a+b\tan^{-1}(cx)\right)\,dx$

Optimal. Leaf size=103

$$\frac{(d+ex)^3\left(a+b\tan^{-1}(cx)\right)}{3e}-\frac{b\left(3c^2d^2-e^2\right)\log\left(c^2x^2+1\right)}{6c^3}-\frac{bd\left(d^2-\frac{3e^2}{c^2}\right)\tan^{-1}(cx)}{3e}-\frac{bdex}{c}-\frac{be^2x^2}{6c}$$

```
[Out] -((b*d*e*x)/c) - (b*e^2*x^2)/(6*c) - (b*d*(d^2 - (3*e^2)/c^2)*ArcTan[c*x])/
(3*e) + ((d + e*x)^3*(a + b*ArcTan[c*x]))/(3*e) - (b*(3*c^2*d^2 - e^2)*Log[
1 + c^2*x^2])/(6*c^3)
```

Rubi [A] time = 0.0896576, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {4862, 702, 635, 203, 260}

$$\frac{(d+ex)^3\left(a+b\tan^{-1}(cx)\right)}{3e}-\frac{b\left(3c^2d^2-e^2\right)\log\left(c^2x^2+1\right)}{6c^3}-\frac{bd\left(d^2-\frac{3e^2}{c^2}\right)\tan^{-1}(cx)}{3e}-\frac{bdex}{c}-\frac{be^2x^2}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] -((b*d*e*x)/c) - (b*e^2*x^2)/(6*c) - (b*d*(d^2 - (3*e^2)/c^2)*ArcTan[c*x])/
(3*e) + ((d + e*x)^3*(a + b*ArcTan[c*x]))/(3*e) - (b*(3*c^2*d^2 - e^2)*Log[
1 + c^2*x^2])/(6*c^3)
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int (d+ex)^2\left(a+b\tan^{-1}(cx)\right)dx &= \frac{(d+ex)^3\left(a+b\tan^{-1}(cx)\right)}{3e} - \frac{(bc)\int \frac{(d+ex)^3}{1+c^2x^2}\,dx}{3e} \\
&= \frac{(d+ex)^3\left(a+b\tan^{-1}(cx)\right)}{3e} - \frac{(bc)\int\left(\frac{3de^2}{c^2}+\frac{e^3x}{c^2}+\frac{c^2d^3-3de^2+e\left(3c^2d^2-e^2\right)x}{c^2\left(1+c^2x^2\right)}\right)dx}{3e} \\
&= -\frac{bdex}{c} - \frac{be^2x^2}{6c} + \frac{(d+ex)^3\left(a+b\tan^{-1}(cx)\right)}{3e} - \frac{b\int \frac{c^2d^3-3de^2+e\left(3c^2d^2-e^2\right)x}{1+c^2x^2}\,dx}{3ce} \\
&= -\frac{bdex}{c} - \frac{be^2x^2}{6c} + \frac{(d+ex)^3\left(a+b\tan^{-1}(cx)\right)}{3e} - \frac{1}{3}\left(bd\left(\frac{cd^2}{e}-\frac{3e}{c}\right)\right)\int \frac{1}{1+c^2x^2}\,dx - \frac{\left(}{} \\
&= -\frac{bdex}{c} - \frac{be^2x^2}{6c} - \frac{bd\left(d^2-\frac{3e^2}{c^2}\right)\tan^{-1}(cx)}{3e} + \frac{(d+ex)^3\left(a+b\tan^{-1}(cx)\right)}{3e} - \frac{b\left(3c^2d^2-c\right.}{}
\end{aligned}$$

Mathematica [A] time = 0.329405, size = 163, normalized size = 1.58

$$\frac{(d+ex)^3\left(a+b\tan^{-1}(cx)\right) - \frac{b\left(\left(c^2d^2\left(\sqrt{-c^2}d+3e\right)-e^2\left(3\sqrt{-c^2}d+e\right)\right)\log\left(1-\sqrt{-c^2}x\right)-\left(c^2d^2\left(\sqrt{-c^2}d-3e\right)+e^2\left(e-3\sqrt{-c^2}d\right)\right)\log\left(\sqrt{-c^2}x+1\right)+c^2e^2x(6d+ex)\right)}{2c^3}}{3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] ((d + e*x)^3*(a + b*ArcTan[c*x]) - (b*(c^2*e^2*x*(6*d + e*x) + (-(e^2*(3*Sq
rt[-c^2]*d + e)) + c^2*d^2*(Sqrt[-c^2]*d + 3*e))*Log[1 - Sqrt[-c^2]*x] - (c
^2*d^2*(Sqrt[-c^2]*d - 3*e) + e^2*(-3*Sqrt[-c^2]*d + e))*Log[1 + Sqrt[-c^2]
*x]))/(2*c^3))/(3*e)
```

Maple [A] time = 0.027, size = 137, normalized size = 1.3

$$\frac{ae^2x^3}{3} + aex^2d + axd^2 + \frac{ad^3}{3e} + \frac{be^2\arctan(cx)x^3}{3} + be\arctan(cx)x^2d + b\arctan(cx)xd^2 - \frac{be^2x^2}{6c} - \frac{bedx}{c} - \frac{b\ln\left(c^2x^2}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(a+b*arctan(c*x)),x)
```

```
[Out] 1/3*a*e^2*x^3+a*e*x^2*d+a*x*d^2+1/3*a/e*d^3+1/3*b*e^2*arctan(c*x)*x^3+b*e*a
rctan(c*x)*x^2*d+b*arctan(c*x)*x*d^2-1/6*b*e^2*x^2/c-b*d*e*x/c-1/2/c*b*ln(c
^2*x^2+1)*d^2+1/6/c^3*b*e^2*ln(c^2*x^2+1)+1/c^2*b*e*arctan(c*x)*d
```

Maxima [A] time = 1.46562, size = 170, normalized size = 1.65

$$\frac{1}{3}ae^2x^3 + adex^2 + \left(x^2\arctan(cx) - c\left(\frac{x}{c^2} - \frac{\arctan(cx)}{c^3}\right)\right)bde + \frac{1}{6}\left(2x^3\arctan(cx) - c\left(\frac{x^2}{c^2} - \frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)be^2 + ad^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*e^2*x^3 + a*d*e*x^2 + (x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))
*b*d*e + 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*e^2
 + a*d^2*x + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d^2/c
```

Fricas [A] time = 2.30074, size = 281, normalized size = 2.73

$$\frac{2ac^3e^2x^3 + \left(6ac^3de - bc^2e^2\right)x^2 + 6\left(ac^3d^2 - bc^2de\right)x + 2\left(bc^3e^2x^3 + 3bc^3dex^2 + 3bc^3d^2x + 3bcde\right)\arctan(cx) - \left(3bc^2d\right.}{6c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*e^2*x^3 + (6*a*c^3*d*e - b*c^2*e^2)*x^2 + 6*(a*c^3*d^2 - b*c^2
*d*e)*x + 2*(b*c^3*e^2*x^3 + 3*b*c^3*d*e*x^2 + 3*b*c^3*d^2*x + 3*b*c*d*e)*a
rctan(c*x) - (3*b*c^2*d^2 - b*e^2)*log(c^2*x^2 + 1))/c^3
```

Sympy [A] time = 1.31008, size = 160, normalized size = 1.55

$$\begin{cases} ad^2x + adex^2 + \frac{ae^2x^3}{3} + bd^2x\operatorname{atan}(cx) + bdex^2\operatorname{atan}(cx) + \frac{be^2x^3\operatorname{atan}(cx)}{3} - \frac{bd^2\log\left(x^2+\frac{1}{c^2}\right)}{2c} - \frac{bdex}{c} - \frac{be^2x^2}{6c} + \frac{bde\operatorname{atan}(cx)}{c^2} + \frac{be^2} \\ a\left(d^2x + dex^2 + \frac{e^2x^3}{3}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*d**2*x + a*d*e*x**2 + a*e**2*x**3/3 + b*d**2*x*atan(c*x) + b*d
*e*x**2*atan(c*x) + b*e**2*x**3*atan(c*x)/3 - b*d**2*log(x**2 + c**(-2))/(2
*c) - b*d*e*x/c - b*e**2*x**2/(6*c) + b*d*e*atan(c*x)/c**2 + b*e**2*log(x**
2 + c**(-2))/(6*c**3), Ne(c, 0)), (a*(d**2*x + d*e*x**2 + e**2*x**3/3), Tru
e))
```

Giac [A] time = 1.13929, size = 217, normalized size = 2.11

$$\frac{2\,bc^3x^3\arctan(cx)\,e^2 + 6\,bc^3dx^2\arctan(cx)\,e + 6\,bc^3d^2x\arctan(cx) + 2\,ac^3x^3e^2 + 6\,ac^3dx^2e + 6\,ac^3d^2x - 6\,\pi bcde\operatorname{sgn}(c}{6\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] 1/6*(2*b*c^3*x^3*arctan(c*x)*e^2 + 6*b*c^3*d*x^2*arctan(c*x)*e + 6*b*c^3*d^
2*x*arctan(c*x) + 2*a*c^3*x^3*e^2 + 6*a*c^3*d*x^2*e + 6*a*c^3*d^2*x - 6*pi*
b*c*d*e*sgn(c)*sgn(x) - b*c^2*x^2*e^2 - 6*b*c^2*d*x*e - 3*b*c^2*d^2*log(c^2
*x^2 + 1) + 6*b*c*d*arctan(c*x)*e + b*e^2*log(c^2*x^2 + 1))/c^3
```

3.4 $\int (d+ex)\left(a+b\tan^{-1}(cx)\right)\,dx$

Optimal. Leaf size=76

$$\frac{(d+ex)^2\left(a+b\tan^{-1}(cx)\right)}{2e}-\frac{b\left(d^2-\frac{e^2}{c^2}\right)\tan^{-1}(cx)}{2e}-\frac{bd\log\left(c^2x^2+1\right)}{2c}-\frac{bex}{2c}$$

```
[Out] -(b*e*x)/(2*c) - (b*(d^2 - e^2/c^2)*ArcTan[c*x])/(2*e) + ((d + e*x)^2*(a +
b*ArcTan[c*x]))/(2*e) - (b*d*Log[1 + c^2*x^2])/(2*c)
```

Rubi [A] time = 0.061302, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4862, 702, 635, 203, 260}

$$\frac{(d+ex)^2\left(a+b\tan^{-1}(cx)\right)}{2e}-\frac{b\left(d^2-\frac{e^2}{c^2}\right)\tan^{-1}(cx)}{2e}-\frac{bd\log\left(c^2x^2+1\right)}{2c}-\frac{bex}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + b*ArcTan[c*x]),x]
```

```
[Out] -(b*e*x)/(2*c) - (b*(d^2 - e^2/c^2)*ArcTan[c*x])/(2*e) + ((d + e*x)^2*(a +
b*ArcTan[c*x]))/(2*e) - (b*d*Log[1 + c^2*x^2])/(2*c)
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int (d+ex)\left(a+b\tan^{-1}(cx)\right)dx &= \frac{(d+ex)^2\left(a+b\tan^{-1}(cx)\right)}{2e} - \frac{(bc)\int \frac{(d+ex)^2}{1+c^2x^2}\,dx}{2e} \\
&= \frac{(d+ex)^2\left(a+b\tan^{-1}(cx)\right)}{2e} - \frac{(bc)\int \left(\frac{e^2}{c^2} + \frac{c^2d^2-e^2+2c^2dex}{c^2\left(1+c^2x^2\right)}\right)dx}{2e} \\
&= -\frac{bex}{2c} + \frac{(d+ex)^2\left(a+b\tan^{-1}(cx)\right)}{2e} - \frac{b\int \frac{c^2d^2-e^2+2c^2dex}{1+c^2x^2}\,dx}{2ce} \\
&= -\frac{bex}{2c} + \frac{(d+ex)^2\left(a+b\tan^{-1}(cx)\right)}{2e} - (bcd)\int \frac{x}{1+c^2x^2}\,dx - \frac{(b(cd-e)(cd+e))\int \frac{1}{1+c^2x}}{2ce} \\
&= -\frac{bex}{2c} - \frac{b\left(d^2-\frac{e^2}{c^2}\right)\tan^{-1}(cx)}{2e} + \frac{(d+ex)^2\left(a+b\tan^{-1}(cx)\right)}{2e} - \frac{bd\log\left(1+c^2x^2\right)}{2c}
\end{aligned}$$

Mathematica [A] time = 0.0054695, size = 77, normalized size = 1.01

$$adx + \frac{1}{2}aex^2 - \frac{bd\log\left(c^2x^2+1\right)}{2c} + \frac{be\tan^{-1}(cx)}{2c^2} + bdx\tan^{-1}(cx) + \frac{1}{2}bex^2\tan^{-1}(cx) - \frac{bex}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + b*ArcTan[c*x]),x]
```

```
[Out] a*d*x - (b*e*x)/(2*c) + (a*e*x^2)/2 + (b*e*ArcTan[c*x])/(2*c^2) + b*d*x*Arc
Tan[c*x] + (b*e*x^2*ArcTan[c*x])/2 - (b*d*Log[1 + c^2*x^2])/(2*c)
```

Maple [A] time = 0.024, size = 68, normalized size = 0.9

$$\frac{ax^2e}{2} + adx + \frac{b\arctan(cx)\,x^2e}{2} + b\arctan(cx)\,xd - \frac{ebx}{2c} - \frac{bd\ln\left(c^2x^2+1\right)}{2c} + \frac{\arctan(cx)\,be}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(a+b*arctan(c*x)),x)
```

```
[Out] 1/2*a*x^2*e+a*d*x+1/2*b*arctan(c*x)*x^2*e+b*arctan(c*x)*x*d-1/2*b*e*x/c-1/2
*b*d*ln(c^2*x^2+1)/c+1/2/c^2*b*e*arctan(c*x)
```

Maxima [A] time = 1.48609, size = 96, normalized size = 1.26

$$\frac{1}{2}aex^2 + \frac{1}{2}\left(x^2\arctan(cx) - c\left(\frac{x}{c^2} - \frac{\arctan(cx)}{c^3}\right)\right)be + adx + \frac{\left(2cx\arctan(cx) - \log\left(c^2x^2+1\right)\right)bd}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/2*a*e*x^2 + 1/2*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b*e + a*d
*x + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d/c
```

Fricas [A] time = 2.11601, size = 162, normalized size = 2.13

$$\frac{ac^2ex^2 - bcd\log\left(c^2x^2+1\right) + \left(2ac^2d - bce\right)x + \left(bc^2ex^2 + 2bc^2dx + be\right)\arctan(cx)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*c^2*e*x^2 - b*c*d*log(c^2*x^2 + 1) + (2*a*c^2*d - b*c*e)*x + (b*c^2*
e*x^2 + 2*b*c^2*d*x + b*e)*arctan(c*x))/c^2
```

Sympy [A] time = 0.730579, size = 87, normalized size = 1.14

$$\begin{cases} adx + \frac{aex^2}{2} + bdx\operatorname{atan}(cx) + \frac{bex^2\operatorname{atan}(cx)}{2} - \frac{bd\log\left(x^2+\frac{1}{c^2}\right)}{2c} - \frac{bex}{2c} + \frac{be\operatorname{atan}(cx)}{2c^2} & \text{for } c \neq 0 \\ a\left(dx + \frac{ex^2}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*d*x + a*e*x**2/2 + b*d*x*atan(c*x) + b*e*x**2*atan(c*x)/2 - b*
d*log(x**2 + c**(-2))/(2*c) - b*e*x/(2*c) + b*e*atan(c*x)/(2*c**2), Ne(c, 0
)), (a*(d*x + e*x**2/2), True))
```

Giac [A] time = 1.22245, size = 122, normalized size = 1.61

$$\frac{bc^2x^2\arctan(cx)\,e + 2\,bc^2dx\arctan(cx) + ac^2x^2e + 2\,ac^2dx - \pi be\operatorname{sgn}(c)\operatorname{sgn}(x) - bcxe - bcd\log\left(c^2x^2+1\right) + b\arctan}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] 1/2*(b*c^2*x^2*arctan(c*x)*e + 2*b*c^2*d*x*arctan(c*x) + a*c^2*x^2*e + 2*a*
c^2*d*x - pi*b*e*sgn(c)*sgn(x) - b*c*x*e - b*c*d*log(c^2*x^2 + 1) + b*arcta
n(c*x)*e)/c^2
```

3.5 $\int \frac{a+b\tan^{-1}(cx)}{d+ex}\,dx$

Optimal. Leaf size=138

$$-\frac{ib\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{2e}+\frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e}+\frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e}-\frac{\log\left(\frac{2}{1-icx}\right)\left(a+b\right.}{e}$$

```
[Out] -(((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e) + ((a + b*ArcTan[c*x])*Log[(2
*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e + ((I/2)*b*PolyLog[2, 1 - 2/(1
- I*c*x)])/e - ((I/2)*b*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*
c*x))])/e
```

Rubi [A] time = 0.0832176, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4856, 2402, 2315, 2447}

$$-\frac{ib\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{2e}+\frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e}+\frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e}-\frac{\log\left(\frac{2}{1-icx}\right)\left(a+b\right.}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(d + e*x),x]
```

```
[Out] -(((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e) + ((a + b*ArcTan[c*x])*Log[(2
*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e + ((I/2)*b*PolyLog[2, 1 - 2/(1
- I*c*x)])/e - ((I/2)*b*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*
c*x))])/e
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{d+ex}\,dx = -\frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{e} + \frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e} + \frac{(bc)\int \frac{\log\left(\frac{2}{1-icx}\right)}{1+c^2x^2}\,dx}{e} - \frac{(bc)}{}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{e} + \frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e} - \frac{ib\,\mathrm{Li}_2\left(1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e} +$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{e} + \frac{\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e} + \frac{ib\,\mathrm{Li}_2\left(1-\frac{2}{1-icx}\right)}{2e} - \frac{ib\,\mathrm{Li}_2}{}$$

Mathematica [A] time = 0.0599263, size = 138, normalized size = 1.

$$\frac{ib\,\mathrm{PolyLog}\left(2,\frac{e(1-icx)}{e+icd}\right) - ib\,\mathrm{PolyLog}\left(2,-\frac{e(cx-i)}{cd+ie}\right) + 2a\log(d+ex) + ib\log(1-icx)\log\left(\frac{c(d+ex)}{cd-ie}\right) - ib\log(1+icx)\log\left(\frac{c(d+e}{cd+}\right.}{2e}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])/(d + e*x),x]
```

```
[Out] (2*a*Log[d + e*x] + I*b*Log[1 - I*c*x]*Log[(c*(d + e*x))/(c*d - I*e)] - I*b
*Log[1 + I*c*x]*Log[(c*(d + e*x))/(c*d + I*e)] + I*b*PolyLog[2, (e*(1 - I*c
*x))/(I*c*d + e)] - I*b*PolyLog[2, -((e*(-I + c*x))/(c*d + I*e))])/(2*e)
```

Maple [A] time = 0.054, size = 168, normalized size = 1.2

$$\frac{a\ln(ecx+dc)}{e}+\frac{b\ln(ecx+dc)\arctan(cx)}{e}+\frac{\frac{i}{2}b\ln(ecx+dc)}{e}\ln\left(\frac{ie-ecx}{dc+ie}\right)-\frac{\frac{i}{2}b\ln(ecx+dc)}{e}\ln\left(\frac{ie+ecx}{ie-dc}\right)+\frac{\frac{i}{2}b}{e}\text{dilog}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/(e*x+d),x)
```

```
[Out] a*ln(c*e*x+c*d)/e+b*ln(c*e*x+c*d)/e*arctan(c*x)+1/2*I*b*ln(c*e*x+c*d)/e*ln(
(I*e-e*c*x)/(d*c+I*e))-1/2*I*b*ln(c*e*x+c*d)/e*ln((I*e+e*c*x)/(I*e-d*c))+1/
2*I*b/e*dilog((I*e-e*c*x)/(d*c+I*e))-1/2*I*b/e*dilog((I*e+e*c*x)/(I*e-d*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$2b\int\frac{\arctan(cx)}{2(ex+d)}\,dx+\frac{a\log(ex+d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x+d),x, algorithm="maxima")
```

```
[Out] 2*b*integrate(1/2*arctan(c*x)/(e*x + d), x) + a*log(e*x + d)/e
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\arctan(cx)+a}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)/(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\operatorname{atan}(cx)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/(e*x+d),x)
```

```
[Out] Integral((a + b*atan(c*x))/(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \arctan(cx) + a}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)/(e*x + d), x)
```

3.6 $\int \frac{a+b\tan^{-1}(cx)}{(d+ex)^2}\,dx$

Optimal. Leaf size=98

$$-\frac{a+b\tan^{-1}(cx)}{e(d+ex)}-\frac{bc\log\left(c^2x^2+1\right)}{2\left(c^2d^2+e^2\right)}+\frac{bc\log(d+ex)}{c^2d^2+e^2}+\frac{bc^2d\tan^{-1}(cx)}{e\left(c^2d^2+e^2\right)}$$

```
[Out] (b*c^2*d*ArcTan[c*x])/(e*(c^2*d^2 + e^2)) - (a + b*ArcTan[c*x])/(e*(d + e*x
)) + (b*c*Log[d + e*x])/(c^2*d^2 + e^2) - (b*c*Log[1 + c^2*x^2])/(2*(c^2*d^
2 + e^2))
```

Rubi [A] time = 0.05334, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4862, 706, 31, 635, 203, 260}

$$-\frac{a+b\tan^{-1}(cx)}{e(d+ex)}-\frac{bc\log\left(c^2x^2+1\right)}{2\left(c^2d^2+e^2\right)}+\frac{bc\log(d+ex)}{c^2d^2+e^2}+\frac{bc^2d\tan^{-1}(cx)}{e\left(c^2d^2+e^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(d + e*x)^2,x]
```

```
[Out] (b*c^2*d*ArcTan[c*x])/(e*(c^2*d^2 + e^2)) - (a + b*ArcTan[c*x])/(e*(d + e*x
)) + (b*c*Log[d + e*x])/(c^2*d^2 + e^2) - (b*c*Log[1 + c^2*x^2])/(2*(c^2*d^
2 + e^2))
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tan^{-1}(cx)}{(d+ex)^2}\,dx &= -\frac{a+b\tan^{-1}(cx)}{e(d+ex)} + \frac{(bc)\int \frac{1}{(d+ex)\left(1+c^2x^2\right)}\,dx}{e} \\
&= -\frac{a+b\tan^{-1}(cx)}{e(d+ex)} + \frac{(bc)\int \frac{c^2d-c^2ex}{1+c^2x^2}\,dx}{e\left(c^2d^2+e^2\right)} + \frac{(bce)\int \frac{1}{d+ex}\,dx}{c^2d^2+e^2} \\
&= -\frac{a+b\tan^{-1}(cx)}{e(d+ex)} + \frac{bc\log(d+ex)}{c^2d^2+e^2} - \frac{\left(bc^3\right)\int \frac{x}{1+c^2x^2}\,dx}{c^2d^2+e^2} + \frac{\left(bc^3d\right)\int \frac{1}{1+c^2x^2}\,dx}{e\left(c^2d^2+e^2\right)} \\
&= \frac{bc^2d\tan^{-1}(cx)}{e\left(c^2d^2+e^2\right)} - \frac{a+b\tan^{-1}(cx)}{e(d+ex)} + \frac{bc\log(d+ex)}{c^2d^2+e^2} - \frac{bc\log\left(1+c^2x^2\right)}{2\left(c^2d^2+e^2\right)}
\end{aligned}$$

Mathematica [A] time = 0.195202, size = 111, normalized size = 1.13

$$\frac{\frac{bc\left(\left(\sqrt{-c^2}d-e\right)\log\left(1-\sqrt{-c^2}x\right)-\left(\sqrt{-c^2}d+e\right)\log\left(\sqrt{-c^2}x+1\right)+2e\log(d+ex)\right)}{2\left(c^2d^2+e^2\right)} - \frac{a+b\tan^{-1}(cx)}{d+ex}}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(d + e*x)^2,x]
```

```
[Out] (-((a + b*ArcTan[c*x])/(d + e*x)) + (b*c*((Sqrt[-c^2]*d - e)*Log[1 - Sqrt[-
c^2]*x] - (Sqrt[-c^2]*d + e)*Log[1 + Sqrt[-c^2]*x] + 2*e*Log[d + e*x]))/(2*
(c^2*d^2 + e^2)))/e
```

Maple [A] time = 0.031, size = 118, normalized size = 1.2

$$-\frac{ac}{(ecx+dc)\,e}-\frac{bc\arctan(cx)}{(ecx+dc)\,e}+\frac{bc\ln(ecx+dc)}{c^2d^2+e^2}-\frac{bc\ln\left(c^2x^2+1\right)}{2\,c^2d^2+2\,e^2}+\frac{bc^2d\arctan(cx)}{e\left(c^2d^2+e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/(e*x+d)^2,x)
```

```
[Out] -c*a/(c*e*x+c*d)/e-c*b/(c*e*x+c*d)/e*arctan(c*x)+c*b/(c^2*d^2+e^2)*ln(c*e*x
+c*d)-1/2*b*c*ln(c^2*x^2+1)/(c^2*d^2+e^2)+b*c^2*d*arctan(c*x)/e/(c^2*d^2+e^
2)
```

Maxima [A] time = 1.48091, size = 144, normalized size = 1.47

$$\frac{1}{2}\left(\left(\frac{2\,cd\arctan(cx)}{c^2d^2e+e^3}-\frac{\log\left(c^2x^2+1\right)}{c^2d^2+e^2}+\frac{2\,\log(ex+d)}{c^2d^2+e^2}\right)c-\frac{2\,\arctan(cx)}{e^2x+de}\right)b-\frac{a}{e^2x+de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((2*c*d*arctan(c*x)/(c^2*d^2*e + e^3) - log(c^2*x^2 + 1)/(c^2*d^2 + e^2
) + 2*log(e*x + d)/(c^2*d^2 + e^2))*c - 2*arctan(c*x)/(e^2*x + d*e))*b - a/
(e^2*x + d*e)
```

Fricas [A] time = 2.50458, size = 259, normalized size = 2.64

$$-\frac{2\,ac^2d^2+2\,ae^2-2\left(bc^2dex-be^2\right)\arctan(cx)+\left(bce^2x+bcde\right)\log\left(c^2x^2+1\right)-2\left(bce^2x+bcde\right)\log(ex+d)}{2\left(c^2d^3e+de^3+\left(c^2d^2e^2+e^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*c^2*d^2 + 2*a*e^2 - 2*(b*c^2*d*e*x - b*e^2)*arctan(c*x) + (b*c*e^
2*x + b*c*d*e)*log(c^2*x^2 + 1) - 2*(b*c*e^2*x + b*c*d*e)*log(e*x + d))/(c^
2*d^3*e + d*e^3 + (c^2*d^2*e^2 + e^4)*x)
```

Sympy [A] time = 46.2556, size = 777, normalized size = 7.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/(e*x+d)**2,x)
```

```
[Out] Piecewise((-2*a*d/(4*d**2*e + 4*d*e**2*x) + 2*a*e*x/(4*d**2*e + 4*d*e**2*x)
 + 2*I*b*d*atanh(e*x/d)/(4*d**2*e + 4*d*e**2*x) + I*b*d/(4*d**2*e + 4*d*e**
2*x) - 2*I*b*e*x*atanh(e*x/d)/(4*d**2*e + 4*d*e**2*x) - I*b*e*x/(4*d**2*e +
 4*d*e**2*x), Eq(c, -I*e/d)), (-2*a*d/(4*d**2*e + 4*d*e**2*x) + 2*a*e*x/(4*
d**2*e + 4*d*e**2*x) - 2*I*b*d*atanh(e*x/d)/(4*d**2*e + 4*d*e**2*x) - I*b*d
/(4*d**2*e + 4*d*e**2*x) + 2*I*b*e*x*atanh(e*x/d)/(4*d**2*e + 4*d*e**2*x) +
 I*b*e*x/(4*d**2*e + 4*d*e**2*x), Eq(c, I*e/d)), (zoo*(a*x + b*x*atan(c*x)
- b*log(x**2 + c**(-2))/(2*c)), Eq(d, -e*x)), ((a*x + b*x*atan(c*x) - b*log
(x**2 + c**(-2))/(2*c))/d**2, Eq(e, 0)), (a*x/(d**2 + d*e*x), Eq(c, 0)), (-
2*a*c**2*d**2/(2*c**2*d**3*e + 2*c**2*d**2*e**2*x + 2*d*e**3 + 2*e**4*x) -
2*a*e**2/(2*c**2*d**3*e + 2*c**2*d**2*e**2*x + 2*d*e**3 + 2*e**4*x) + 2*b*c
**2*d*e*x*atan(c*x)/(2*c**2*d**3*e + 2*c**2*d**2*e**2*x + 2*d*e**3 + 2*e**4
*x) - b*c*d*e*log(x**2 + c**(-2))/(2*c**2*d**3*e + 2*c**2*d**2*e**2*x + 2*d
*e**3 + 2*e**4*x) + 2*b*c*d*e*log(d/e + x)/(2*c**2*d**3*e + 2*c**2*d**2*e**
2*x + 2*d*e**3 + 2*e**4*x) - b*c*e**2*x*log(x**2 + c**(-2))/(2*c**2*d**3*e
+ 2*c**2*d**2*e**2*x + 2*d*e**3 + 2*e**4*x) + 2*b*c*e**2*x*log(d/e + x)/(2*
c**2*d**3*e + 2*c**2*d**2*e**2*x + 2*d*e**3 + 2*e**4*x) - 2*b*e**2*atan(c*x
)/(2*c**2*d**3*e + 2*c**2*d**2*e**2*x + 2*d*e**3 + 2*e**4*x), True))
```

Giac [A] time = 1.14614, size = 217, normalized size = 2.21

$$\frac{1}{2}\left(\left(\frac{2\,cd\arctan\left(\frac{\left(c^2d-\frac{c^2d^2}{xe+d}-\frac{e^2}{xe+d}\right)e^{(-1)}}{c}\right)e^{(-2)}}{c^2d^2e+e^3}-\frac{\log\left(c^2-\frac{2\,c^2d}{xe+d}+\frac{c^2d^2}{(xe+d)^2}+\frac{e^2}{(xe+d)^2}\right)}{c^2d^2e^2+e^4}\right)ce^2-\frac{2\,\arctan\left(cx\right)e^{(-1)}}{xe+d}\right)b-\frac{ae^{(-1)}}{xe+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/2*((2*c*d*arctan((c^2*d - c^2*d^2/(x*e + d) - e^2/(x*e + d))*e^(-1)/c)*e^
(-2)/(c^2*d^2*e + e^3) - log(c^2 - 2*c^2*d/(x*e + d) + c^2*d^2/(x*e + d)^2
+ e^2/(x*e + d)^2)/(c^2*d^2*e^2 + e^4))*c*e^2 - 2*arctan(c*x)*e^(-1)/(x*e +
 d))*b - a*e^(-1)/(x*e + d)
```

3.7 $\int \frac{a+b\tan^{-1}(cx)}{(d+ex)^3}\,dx$

Optimal. Leaf size=146

$$-\frac{a+b\tan^{-1}(cx)}{2e(d+ex)^2}-\frac{bc^3d\log\left(c^2x^2+1\right)}{2\left(c^2d^2+e^2\right)^2}-\frac{bc}{2\left(c^2d^2+e^2\right)(d+ex)}+\frac{bc^3d\log(d+ex)}{\left(c^2d^2+e^2\right)^2}+\frac{bc^2(cd-e)(cd+e)\tan^{-1}(cx)}{2e\left(c^2d^2+e^2\right)^2}$$

```
[Out] -(b*c)/(2*(c^2*d^2 + e^2)*(d + e*x)) + (b*c^2*(c*d - e)*(c*d + e)*ArcTan[c*
x])/(2*e*(c^2*d^2 + e^2)^2) - (a + b*ArcTan[c*x])/(2*e*(d + e*x)^2) + (b*c^
3*d*Log[d + e*x])/(c^2*d^2 + e^2)^2 - (b*c^3*d*Log[1 + c^2*x^2])/(2*(c^2*d^
2 + e^2)^2)
```

Rubi [A] time = 0.123111, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4862, 710, 801, 635, 203, 260}

$$-\frac{a+b\tan^{-1}(cx)}{2e(d+ex)^2}-\frac{bc^3d\log\left(c^2x^2+1\right)}{2\left(c^2d^2+e^2\right)^2}-\frac{bc}{2\left(c^2d^2+e^2\right)(d+ex)}+\frac{bc^3d\log(d+ex)}{\left(c^2d^2+e^2\right)^2}+\frac{bc^2(cd-e)(cd+e)\tan^{-1}(cx)}{2e\left(c^2d^2+e^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(d + e*x)^3,x]
```

```
[Out] -(b*c)/(2*(c^2*d^2 + e^2)*(d + e*x)) + (b*c^2*(c*d - e)*(c*d + e)*ArcTan[c*
x])/(2*e*(c^2*d^2 + e^2)^2) - (a + b*ArcTan[c*x])/(2*e*(d + e*x)^2) + (b*c^
3*d*Log[d + e*x])/(c^2*d^2 + e^2)^2 - (b*c^3*d*Log[1 + c^2*x^2])/(2*(c^2*d^
2 + e^2)^2)
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), In
t[((d + e*x)^(m + 1)*(d - e*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m
```

```
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{(d+ex)^3}\,dx = -\frac{a+b\tan^{-1}(cx)}{2e(d+ex)^2} + \frac{(bc)\int \frac{1}{(d+ex)^2\left(1+c^2x^2\right)}\,dx}{2e}$$

$$= -\frac{bc}{2\left(c^2d^2+e^2\right)(d+ex)} - \frac{a+b\tan^{-1}(cx)}{2e(d+ex)^2} + \frac{\left(bc^3\right)\int \frac{d-ex}{(d+ex)\left(1+c^2x^2\right)}\,dx}{2e\left(c^2d^2+e^2\right)}$$

$$= -\frac{bc}{2\left(c^2d^2+e^2\right)(d+ex)} - \frac{a+b\tan^{-1}(cx)}{2e(d+ex)^2} + \frac{\left(bc^3\right)\int \left(\frac{2de^2}{\left(c^2d^2+e^2\right)(d+ex)} + \frac{c^2d^2-e^2-2c^2dex}{\left(c^2d^2+e^2\right)\left(1+c^2x^2\right)}\right)dx}{2e\left(c^2d^2+e^2\right)}$$

$$= -\frac{bc}{2\left(c^2d^2+e^2\right)(d+ex)} - \frac{a+b\tan^{-1}(cx)}{2e(d+ex)^2} + \frac{bc^3d\log(d+ex)}{\left(c^2d^2+e^2\right)^2} + \frac{\left(bc^3\right)\int \frac{c^2d^2-e^2-2c^2dex}{1+c^2x^2}\,dx}{2e\left(c^2d^2+e^2\right)^2}$$

$$= -\frac{bc}{2\left(c^2d^2+e^2\right)(d+ex)} - \frac{a+b\tan^{-1}(cx)}{2e(d+ex)^2} + \frac{bc^3d\log(d+ex)}{\left(c^2d^2+e^2\right)^2} - \frac{\left(bc^5d\right)\int \frac{x}{1+c^2x^2}\,dx}{\left(c^2d^2+e^2\right)^2} + \frac{\left(bc^3(cd-e)\right.}{2e}$$

$$= -\frac{bc}{2\left(c^2d^2+e^2\right)(d+ex)} + \frac{bc^2(cd-e)(cd+e)\tan^{-1}(cx)}{2e\left(c^2d^2+e^2\right)^2} - \frac{a+b\tan^{-1}(cx)}{2e(d+ex)^2} + \frac{bc^3d\log(d+ex)}{\left(c^2d^2+e^2\right)^2} - \frac{b}{}$$

Mathematica [A] time = 0.327461, size = 192, normalized size = 1.32

$$-\frac{2\left(a+b\tan^{-1}(cx)\right) + \frac{bc(d+ex)\left(2e\left(c^2d^2+e^2\right)-\left(c^2d\left(\sqrt{-c^2}d-2e\right)-\sqrt{-c^2}e^2\right)\log\left(1-\sqrt{-c^2}x\right)(d+ex)-\left(\sqrt{-c^2}e^2-c^2d\left(\sqrt{-c^2}d+2e\right)\right)\log\left(\sqrt{-c^2}x+1\right)(d+ex)-4c^2de(\right.}{\left(c^2d^2+e^2\right)^2}}{4e(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(d + e*x)^3,x]
```

```
[Out] -(2*(a + b*ArcTan[c*x]) + (b*c*(d + e*x)*(2*e*(c^2*d^2 + e^2) - (c^2*d*(Sqr
t[-c^2]*d - 2*e) - Sqrt[-c^2]*e^2)*(d + e*x)*Log[1 - Sqrt[-c^2]*x] - (Sqrt[
-c^2]*e^2 - c^2*d*(Sqrt[-c^2]*d + 2*e))*(d + e*x)*Log[1 + Sqrt[-c^2]*x] - 4
*c^2*d*e*(d + e*x)*Log[d + e*x]))/(c^2*d^2 + e^2)^2)/(4*e*(d + e*x)^2)
```

Maple [A] time = 0.034, size = 184, normalized size = 1.3

$$-\frac{c^2a}{2\left(ecx+dc\right)^2e} - \frac{c^2b\arctan(cx)}{2\left(ecx+dc\right)^2e} - \frac{c^2b}{\left(2c^2d^2+2e^2\right)(ecx+dc)} + \frac{bc^3d\ln(ecx+dc)}{\left(c^2d^2+e^2\right)^2} + \frac{bc^4\arctan(cx)d^2}{2e\left(c^2d^2+e^2\right)^2} - \frac{bc^3d\ln\left(c^2x^2\right.}{2\left(c^2d^2+e^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/(e*x+d)^3,x)
```

```
[Out] -1/2*c^2*a/(c*e*x+c*d)^2/e-1/2*c^2*b/(c*e*x+c*d)^2/e*arctan(c*x)-1/2*c^2*b/
(c^2*d^2+e^2)/(c*e*x+c*d)+c^3*b*d/(c^2*d^2+e^2)^2*ln(c*e*x+c*d)+1/2*c^4*b/e
/(c^2*d^2+e^2)^2*arctan(c*x)*d^2-1/2*b*c^3*d*ln(c^2*x^2+1)/(c^2*d^2+e^2)^2-
1/2*c^2*b*e/(c^2*d^2+e^2)^2*arctan(c*x)
```

Maxima [A] time = 1.46999, size = 289, normalized size = 1.98

$$-\frac{1}{2}\left(\left(\frac{c^2 d\log\left(c^2x^2+1\right)}{c^4d^4+2c^2d^2e^2+e^4}-\frac{2c^2d\log(ex+d)}{c^4d^4+2c^2d^2e^2+e^4}-\frac{\left(c^4d^2-c^2e^2\right)\arctan(cx)}{\left(c^4d^4e+2c^2d^2e^3+e^5\right)c}+\frac{1}{c^2d^3+de^2+\left(c^2d^2e+e^3\right)x}\right)c+\frac{\arctan}{e^3x^2+2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/2*((c^2*d*log(c^2*x^2 + 1)/(c^4*d^4 + 2*c^2*d^2*e^2 + e^4) - 2*c^2*d*log
(e*x + d)/(c^4*d^4 + 2*c^2*d^2*e^2 + e^4) - (c^4*d^2 - c^2*e^2)*arctan(c*x)
/((c^4*d^4*e + 2*c^2*d^2*e^3 + e^5)*c) + 1/(c^2*d^3 + d*e^2 + (c^2*d^2*e +
e^3)*x))*c + arctan(c*x)/(e^3*x^2 + 2*d*e^2*x + d^2*e))*b - 1/2*a/(e^3*x^2
+ 2*d*e^2*x + d^2*e)
```

Fricas [B] time = 3.64017, size = 637, normalized size = 4.36

$$-\frac{ac^4d^4+bc^3d^3e+2ac^2d^2e^2+bcde^3+ae^4+\left(bc^3d^2e^2+bce^4\right)x+\left(3bc^2d^2e^2+be^4-\left(bc^4d^2e^2-bc^2e^4\right)x^2-2\left(bc^4d^3e-b\right.\right.}{2\left(c^4d^6e+2c^2d^4e^3+d^2e^5+\left(c^4d^4e^3+2c^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(a*c^4*d^4 + b*c^3*d^3*e + 2*a*c^2*d^2*e^2 + b*c*d*e^3 + a*e^4 + (b*c^
3*d^2*e^2 + b*c*e^4)*x + (3*b*c^2*d^2*e^2 + b*e^4 - (b*c^4*d^2*e^2 - b*c^2*
e^4)*x^2 - 2*(b*c^4*d^3*e - b*c^2*d*e^3)*x)*arctan(c*x) + (b*c^3*d*e^3*x^2
+ 2*b*c^3*d^2*e^2*x + b*c^3*d^3*e)*log(c^2*x^2 + 1) - 2*(b*c^3*d*e^3*x^2 +
2*b*c^3*d^2*e^2*x + b*c^3*d^3*e)*log(e*x + d))/(c^4*d^6*e + 2*c^2*d^4*e^3 +
```

```
 d^2*e^5 + (c^4*d^4*e^3 + 2*c^2*d^2*e^5 + e^7)*x^2 + 2*(c^4*d^5*e^2 + 2*c^2
*d^3*e^4 + d*e^6)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.63408, size = 489, normalized size = 3.35

$$\frac{bc^4d^2x^2\arctan(cx)e^2 + 2bc^4d^3x\arctan(cx)e - ac^4d^4 - bc^3dx^2e^3\log\left(c^2x^2+1\right) - 2bc^3d^2xe^2\log\left(c^2x^2+1\right) - bc^3d^3e\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] 1/2*(b*c^4*d^2*x^2*arctan(c*x)*e^2 + 2*b*c^4*d^3*x*arctan(c*x)*e - a*c^4*d^
4 - b*c^3*d*x^2*e^3*log(c^2*x^2 + 1) - 2*b*c^3*d^2*x*e^2*log(c^2*x^2 + 1) -
 b*c^3*d^3*e*log(c^2*x^2 + 1) + 2*b*c^3*d*x^2*e^3*log(abs(x*e + d)) + 4*b*c
^3*d^2*x*e^2*log(abs(x*e + d)) + 2*b*c^3*d^3*e*log(abs(x*e + d)) - b*c^3*d^
2*x*e^2 - b*c^3*d^3*e - b*c^2*x^2*arctan(c*x)*e^4 - 2*b*c^2*d*x*arctan(c*x)
*e^3 - 3*b*c^2*d^2*arctan(c*x)*e^2 - 2*a*c^2*d^2*e^2 - b*c*x*e^4 - b*c*d*e^
3 - b*arctan(c*x)*e^4 - a*e^4)/(c^4*d^4*x^2*e^3 + 2*c^4*d^5*x*e^2 + c^4*d^6
*e + 2*c^2*d^2*x^2*e^5 + 4*c^2*d^3*x*e^4 + 2*c^2*d^4*e^3 + x^2*e^7 + 2*d*x*
e^6 + d^2*e^5)
```

3.8 $\int \frac{a+b\tan^{-1}(cx)}{(d+ex)^4}\,dx$

Optimal. Leaf size=206

$$-\frac{a+b\tan^{-1}(cx)}{3e(d+ex)^3}-\frac{bc^3\left(3c^2d^2-e^2\right)\log\left(c^2x^2+1\right)}{6\left(c^2d^2+e^2\right)^3}-\frac{2bc^3d}{3\left(c^2d^2+e^2\right)^2(d+ex)}-\frac{bc}{6\left(c^2d^2+e^2\right)(d+ex)^2}+\frac{bc^3\left(3c^2d^2-e^2\right)}{3\left(c^2d^2+$$

```
[Out] -(b*c)/(6*(c^2*d^2 + e^2)*(d + e*x)^2) - (2*b*c^3*d)/(3*(c^2*d^2 + e^2)^2*(
d + e*x)) + (b*c^4*d*(c^2*d^2 - 3*e^2)*ArcTan[c*x])/(3*e*(c^2*d^2 + e^2)^3)
 - (a + b*ArcTan[c*x])/(3*e*(d + e*x)^3) + (b*c^3*(3*c^2*d^2 - e^2)*Log[d +
 e*x])/(3*(c^2*d^2 + e^2)^3) - (b*c^3*(3*c^2*d^2 - e^2)*Log[1 + c^2*x^2])/(
6*(c^2*d^2 + e^2)^3)
```

Rubi [A] time = 0.182223, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4862, 710, 801, 635, 203, 260}

$$-\frac{a+b\tan^{-1}(cx)}{3e(d+ex)^3}-\frac{bc^3\left(3c^2d^2-e^2\right)\log\left(c^2x^2+1\right)}{6\left(c^2d^2+e^2\right)^3}-\frac{2bc^3d}{3\left(c^2d^2+e^2\right)^2(d+ex)}-\frac{bc}{6\left(c^2d^2+e^2\right)(d+ex)^2}+\frac{bc^3\left(3c^2d^2-e^2\right)}{3\left(c^2d^2+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/(d + e*x)^4,x]
```

```
[Out] -(b*c)/(6*(c^2*d^2 + e^2)*(d + e*x)^2) - (2*b*c^3*d)/(3*(c^2*d^2 + e^2)^2*(
d + e*x)) + (b*c^4*d*(c^2*d^2 - 3*e^2)*ArcTan[c*x])/(3*e*(c^2*d^2 + e^2)^3)
 - (a + b*ArcTan[c*x])/(3*e*(d + e*x)^3) + (b*c^3*(3*c^2*d^2 - e^2)*Log[d +
 e*x])/(3*(c^2*d^2 + e^2)^3) - (b*c^3*(3*c^2*d^2 - e^2)*Log[1 + c^2*x^2])/(
6*(c^2*d^2 + e^2)^3)
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), In
t[((d + e*x)^(m + 1)*(d - e*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{(d+ex)^4}\,dx = -\frac{a+b\tan^{-1}(cx)}{3e(d+ex)^3} + \frac{(bc)\int \frac{1}{(d+ex)^3\left(1+c^2x^2\right)}\,dx}{3e}$$

$$= -\frac{bc}{6\left(c^2d^2+e^2\right)(d+ex)^2} - \frac{a+b\tan^{-1}(cx)}{3e(d+ex)^3} + \frac{\left(bc^3\right)\int \frac{d-ex}{(d+ex)^2\left(1+c^2x^2\right)}\,dx}{3e\left(c^2d^2+e^2\right)}$$

$$= -\frac{bc}{6\left(c^2d^2+e^2\right)(d+ex)^2} - \frac{a+b\tan^{-1}(cx)}{3e(d+ex)^3} + \frac{\left(bc^3\right)\int\left(\frac{2de^2}{\left(c^2d^2+e^2\right)(d+ex)^2} - \frac{e^2\left(-3c^2d^2+e^2\right)}{\left(c^2d^2+e^2\right)^2(d+ex)} + \frac{c^2d\left(c^2d^2-\right.}{\left(c^2d\right.}\right.}{3e\left(c^2d^2+e^2\right)}$$

$$= -\frac{bc}{6\left(c^2d^2+e^2\right)(d+ex)^2} - \frac{2bc^3d}{3\left(c^2d^2+e^2\right)^2(d+ex)} - \frac{a+b\tan^{-1}(cx)}{3e(d+ex)^3} + \frac{bc^3\left(3c^2d^2-e^2\right)\log(d+}{3\left(c^2d^2+e^2\right)^3}$$

$$= -\frac{bc}{6\left(c^2d^2+e^2\right)(d+ex)^2} - \frac{2bc^3d}{3\left(c^2d^2+e^2\right)^2(d+ex)} - \frac{a+b\tan^{-1}(cx)}{3e(d+ex)^3} + \frac{bc^3\left(3c^2d^2-e^2\right)\log(d+}{3\left(c^2d^2+e^2\right)^3}$$

$$= -\frac{bc}{6\left(c^2d^2+e^2\right)(d+ex)^2} - \frac{2bc^3d}{3\left(c^2d^2+e^2\right)^2(d+ex)} + \frac{bc^4d\left(c^2d^2-3e^2\right)\tan^{-1}(cx)}{3e\left(c^2d^2+e^2\right)^3} - \frac{a+b\tan^{-1}(}{3e(d+ex)}$$

Mathematica [A] time = 0.640897, size = 254, normalized size = 1.23

$$-\frac{2\left(a+b\tan^{-1}(cx)\right) + \frac{bc(d+ex)\left(4c^2de\left(c^2d^2+e^2\right)(d+ex)-c^2\left(c^2d^2\left(\sqrt{-c^2}d-3e\right)+e^2\left(e-3\sqrt{-c^2}d\right)\right)\log\left(1-\sqrt{-c^2}x\right)(d+ex)^2-c^2\left(e^2\left(3\sqrt{-c^2}d+e\right)-c^2d^2\left(\sqrt{-c^2}d+\right.\right.\right.}{\left(c^2d^2+e^2\right)^3}}{6e(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/(d + e*x)^4,x]
```

```
[Out] -(2*(a + b*ArcTan[c*x]) + (b*c*(d + e*x)*(e*(c^2*d^2 + e^2)^2 + 4*c^2*d*e*(
c^2*d^2 + e^2)*(d + e*x) - c^2*(c^2*d^2*(Sqrt[-c^2]*d - 3*e) + e^2*(-3*Sqrt
[-c^2]*d + e))*(d + e*x)^2*Log[1 - Sqrt[-c^2]*x] - c^2*(e^2*(3*Sqrt[-c^2]*d
 + e) - c^2*d^2*(Sqrt[-c^2]*d + 3*e))*(d + e*x)^2*Log[1 + Sqrt[-c^2]*x] - 2
*c^2*e*(3*c^2*d^2 - e^2)*(d + e*x)^2*Log[d + e*x]))/(c^2*d^2 + e^2)^3)/(6*e
*(d + e*x)^3)
```

Maple [A] time = 0.036, size = 282, normalized size = 1.4

$$-\frac{ac^3}{3\,(ecx+dc)^3\,e}-\frac{c^3b\arctan(cx)}{3\,(ecx+dc)^3\,e}-\frac{c^3b}{\left(6\,c^2d^2+6\,e^2\right)(ecx+dc)^2}+\frac{c^5b\ln(ecx+dc)\,d^2}{\left(c^2d^2+e^2\right)^3}-\frac{c^3be^2\ln(ecx+dc)}{3\,\left(c^2d^2+e^2\right)^3}-\frac{2}{3\,\left(c^2d^2+e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/(e*x+d)^4,x)
```

```
[Out] -1/3*c^3*a/(c*e*x+c*d)^3/e-1/3*c^3*b/(c*e*x+c*d)^3/e*arctan(c*x)-1/6*c^3*b/
(c^2*d^2+e^2)/(c*e*x+c*d)^2+c^5*b/(c^2*d^2+e^2)^3*ln(c*e*x+c*d)*d^2-1/3*c^3
*b*e^2/(c^2*d^2+e^2)^3*ln(c*e*x+c*d)-2/3*c^4*b*d/(c^2*d^2+e^2)^2/(c*e*x+c*d
)-1/2*c^5*b/(c^2*d^2+e^2)^3*ln(c^2*x^2+1)*d^2+1/6*c^3*b*e^2/(c^2*d^2+e^2)^3
*ln(c^2*x^2+1)+1/3*c^6*b/e/(c^2*d^2+e^2)^3*arctan(c*x)*d^3-c^4*b*e/(c^2*d^2
+e^2)^3*arctan(c*x)*d
```

Maxima [A] time = 1.55636, size = 505, normalized size = 2.45

$$-\frac{1}{6}\left(c\left(\frac{\left(3\,c^4d^2-c^2e^2\right)\log\left(c^2x^2+1\right)}{c^6d^6+3\,c^4d^4e^2+3\,c^2d^2e^4+e^6}-\frac{2\left(3\,c^4d^2-c^2e^2\right)\log(ex+d)}{c^6d^6+3\,c^4d^4e^2+3\,c^2d^2e^4+e^6}+\frac{4\,c^2dex+5\,c^2d^2}{c^4d^6+2\,c^2d^4e^2+d^2e^4+\left(c^4d^4e^2+2\,c^2d^2e^4+e\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(c*((3*c^4*d^2 - c^2*e^2)*log(c^2*x^2 + 1)/(c^6*d^6 + 3*c^4*d^4*e^2 +
3*c^2*d^2*e^4 + e^6) - 2*(3*c^4*d^2 - c^2*e^2)*log(e*x + d)/(c^6*d^6 + 3*c^
4*d^4*e^2 + 3*c^2*d^2*e^4 + e^6) + (4*c^2*d*e*x + 5*c^2*d^2 + e^2)/(c^4*d^6
 + 2*c^2*d^4*e^2 + d^2*e^4 + (c^4*d^4*e^2 + 2*c^2*d^2*e^4 + e^6)*x^2 + 2*(c
^4*d^5*e + 2*c^2*d^3*e^3 + d*e^5)*x) - 2*(c^6*d^3 - 3*c^4*d*e^2)*arctan(c*x
)/((c^6*d^6*e + 3*c^4*d^4*e^3 + 3*c^2*d^2*e^5 + e^7)*c)) + 2*arctan(c*x)/(e
^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e^2*x + d^3*e))*b - 1/3*a/(e^4*x^3 + 3*d*e^3*x
^2 + 3*d^2*e^2*x + d^3*e)
```

Fricas [B] time = 8.10537, size = 1280, normalized size = 6.21

$$-\frac{2\,ac^6d^6+5\,bc^5d^5e+6\,ac^4d^4e^2+6\,bc^3d^3e^3+6\,ac^2d^2e^4+bcde^5+2\,ae^6+4\left(bc^5d^3e^3+bc^3de^5\right)x^2+\left(9\,bc^5d^4e^2+10\,bc^3d^2e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(2*a*c^6*d^6 + 5*b*c^5*d^5*e + 6*a*c^4*d^4*e^2 + 6*b*c^3*d^3*e^3 + 6*a
*c^2*d^2*e^4 + b*c*d*e^5 + 2*a*e^6 + 4*(b*c^5*d^3*e^3 + b*c^3*d*e^5)*x^2 +
(9*b*c^5*d^4*e^2 + 10*b*c^3*d^2*e^4 + b*c*e^6)*x + 2*(6*b*c^4*d^4*e^2 + 3*b
*c^2*d^2*e^4 + b*e^6 - (b*c^6*d^3*e^3 - 3*b*c^4*d*e^5)*x^3 - 3*(b*c^6*d^4*e
^2 - 3*b*c^4*d^2*e^4)*x^2 - 3*(b*c^6*d^5*e - 3*b*c^4*d^3*e^3)*x)*arctan(c*x
) + (3*b*c^5*d^5*e - b*c^3*d^3*e^3 + (3*b*c^5*d^2*e^4 - b*c^3*e^6)*x^3 + 3*
(3*b*c^5*d^3*e^3 - b*c^3*d*e^5)*x^2 + 3*(3*b*c^5*d^4*e^2 - b*c^3*d^2*e^4)*x
)*log(c^2*x^2 + 1) - 2*(3*b*c^5*d^5*e - b*c^3*d^3*e^3 + (3*b*c^5*d^2*e^4 -
b*c^3*e^6)*x^3 + 3*(3*b*c^5*d^3*e^3 - b*c^3*d*e^5)*x^2 + 3*(3*b*c^5*d^4*e^2
 - b*c^3*d^2*e^4)*x)*log(e*x + d))/(c^6*d^9*e + 3*c^4*d^7*e^3 + 3*c^2*d^5*e
^5 + d^3*e^7 + (c^6*d^6*e^4 + 3*c^4*d^4*e^6 + 3*c^2*d^2*e^8 + e^10)*x^3 + 3
*(c^6*d^7*e^3 + 3*c^4*d^5*e^5 + 3*c^2*d^3*e^7 + d*e^9)*x^2 + 3*(c^6*d^8*e^2
 + 3*c^4*d^6*e^4 + 3*c^2*d^4*e^6 + d^2*e^8)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/(e*x+d)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 10.9004, size = 1042, normalized size = 5.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] 1/6*(2*b*c^6*d^3*x^3*arctan(c*x)*e^3 + 6*b*c^6*d^4*x^2*arctan(c*x)*e^2 + 6*
b*c^6*d^5*x*arctan(c*x)*e - 2*a*c^6*d^6 - 3*b*c^5*d^2*x^3*e^4*log(c^2*x^2 +
 1) - 9*b*c^5*d^3*x^2*e^3*log(c^2*x^2 + 1) - 9*b*c^5*d^4*x*e^2*log(c^2*x^2
```

```
+ 1) - 3*b*c^5*d^5*e*log(c^2*x^2 + 1) + 6*b*c^5*d^2*x^3*e^4*log(abs(x*e + d
)) + 18*b*c^5*d^3*x^2*e^3*log(abs(x*e + d)) + 18*b*c^5*d^4*x*e^2*log(abs(x*
e + d)) + 6*b*c^5*d^5*e*log(abs(x*e + d)) - 4*b*c^5*d^3*x^2*e^3 - 9*b*c^5*d
^4*x*e^2 - 5*b*c^5*d^5*e - 6*b*c^4*d*x^3*arctan(c*x)*e^5 - 18*b*c^4*d^2*x^2
*arctan(c*x)*e^4 - 18*b*c^4*d^3*x*arctan(c*x)*e^3 - 12*b*c^4*d^4*arctan(c*x
)*e^2 - 6*a*c^4*d^4*e^2 + b*c^3*x^3*e^6*log(c^2*x^2 + 1) + 3*b*c^3*d*x^2*e^
5*log(c^2*x^2 + 1) + 3*b*c^3*d^2*x*e^4*log(c^2*x^2 + 1) + b*c^3*d^3*e^3*log
(c^2*x^2 + 1) - 2*b*c^3*x^3*e^6*log(abs(x*e + d)) - 6*b*c^3*d*x^2*e^5*log(a
bs(x*e + d)) - 6*b*c^3*d^2*x*e^4*log(abs(x*e + d)) - 2*b*c^3*d^3*e^3*log(ab
s(x*e + d)) - 4*b*c^3*d*x^2*e^5 - 10*b*c^3*d^2*x*e^4 - 6*b*c^3*d^3*e^3 - 6*
b*c^2*d^2*arctan(c*x)*e^4 - 6*a*c^2*d^2*e^4 - b*c*x*e^6 - b*c*d*e^5 - 2*b*a
rctan(c*x)*e^6 - 2*a*e^6)/(c^6*d^6*x^3*e^4 + 3*c^6*d^7*x^2*e^3 + 3*c^6*d^8*
x*e^2 + c^6*d^9*e + 3*c^4*d^4*x^3*e^6 + 9*c^4*d^5*x^2*e^5 + 9*c^4*d^6*x*e^4
 + 3*c^4*d^7*e^3 + 3*c^2*d^2*x^3*e^8 + 9*c^2*d^3*x^2*e^7 + 9*c^2*d^4*x*e^6
+ 3*c^2*d^5*e^5 + x^3*e^10 + 3*d*x^2*e^9 + 3*d^2*x*e^8 + d^3*e^7)
```

3.9 $\int (d+ex)^3 \left(a+b\tan^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=376

$$\frac{ib^2d(cd-e)(cd+e)\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^3}-\frac{abex\left(6c^2d^2-e^2\right)}{2c^3}-\frac{\left(-6c^2d^2e^2+c^4d^4+e^4\right)\left(a+b\tan^{-1}(cx)\right)^2}{4c^4e}+\frac{id(cd-e$$

```
[Out] (b^2*d*e^2*x)/c^2 - (a*b*e*(6*c^2*d^2 - e^2)*x)/(2*c^3) + (b^2*e^3*x^2)/(12
*c^2) - (b^2*d*e^2*ArcTan[c*x])/c^3 - (b^2*e*(6*c^2*d^2 - e^2)*x*ArcTan[c*x
])/(2*c^3) - (b*d*e^2*x^2*(a + b*ArcTan[c*x]))/c - (b*e^3*x^3*(a + b*ArcTan
[c*x]))/(6*c) + (I*d*(c*d - e)*(c*d + e)*(a + b*ArcTan[c*x])^2)/c^3 - ((c^4
*d^4 - 6*c^2*d^2*e^2 + e^4)*(a + b*ArcTan[c*x])^2)/(4*c^4*e) + ((d + e*x)^4
*(a + b*ArcTan[c*x])^2)/(4*e) + (2*b*d*(c*d - e)*(c*d + e)*(a + b*ArcTan[c*
x])*Log[2/(1 + I*c*x)])/c^3 - (b^2*e^3*Log[1 + c^2*x^2])/(12*c^4) + (b^2*e*
(6*c^2*d^2 - e^2)*Log[1 + c^2*x^2])/(4*c^4) + (I*b^2*d*(c*d - e)*(c*d + e)*
PolyLog[2, 1 - 2/(1 + I*c*x)])/c^3
```

Rubi [A] time = 0.573163, antiderivative size = 376, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 14, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {4864, 4846, 260, 4852, 321, 203, 266, 43, 4984, 4884, 4920, 4854, 2402, 2315}

$$\frac{ib^2d(cd-e)(cd+e)\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^3}-\frac{abex\left(6c^2d^2-e^2\right)}{2c^3}-\frac{\left(-6c^2d^2e^2+c^4d^4+e^4\right)\left(a+b\tan^{-1}(cx)\right)^2}{4c^4e}+\frac{id(cd-e$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (b^2*d*e^2*x)/c^2 - (a*b*e*(6*c^2*d^2 - e^2)*x)/(2*c^3) + (b^2*e^3*x^2)/(12
*c^2) - (b^2*d*e^2*ArcTan[c*x])/c^3 - (b^2*e*(6*c^2*d^2 - e^2)*x*ArcTan[c*x
])/(2*c^3) - (b*d*e^2*x^2*(a + b*ArcTan[c*x]))/c - (b*e^3*x^3*(a + b*ArcTan
[c*x]))/(6*c) + (I*d*(c*d - e)*(c*d + e)*(a + b*ArcTan[c*x])^2)/c^3 - ((c^4
*d^4 - 6*c^2*d^2*e^2 + e^4)*(a + b*ArcTan[c*x])^2)/(4*c^4*e) + ((d + e*x)^4
*(a + b*ArcTan[c*x])^2)/(4*e) + (2*b*d*(c*d - e)*(c*d + e)*(a + b*ArcTan[c*
x])*Log[2/(1 + I*c*x)])/c^3 - (b^2*e^3*Log[1 + c^2*x^2])/(12*c^4) + (b^2*e*
(6*c^2*d^2 - e^2)*Log[1 + c^2*x^2])/(4*c^4) + (I*b^2*d*(c*d - e)*(c*d + e)*
PolyLog[2, 1 - 2/(1 + I*c*x)])/c^3
```

Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4984

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int (d+ex)^3 \left(a+b\tan^{-1}(cx)\right)^2 \, dx = \frac{(d+ex)^4 \left(a+b\tan^{-1}(cx)\right)^2}{4e} - \frac{(bc)\int \left(\frac{e^2\left(6c^2d^2-e^2\right)\left(a+b\tan^{-1}(cx)\right)}{c^4} + \frac{4de^3x\left(a+b\tan^{-1}(cx)\right)}{c^2} + \cdots\right.}{}$$

$$= \frac{(d+ex)^4 \left(a+b\tan^{-1}(cx)\right)^2}{4e} - \frac{b\int \frac{\left(c^4d^4-6c^2d^2e^2+e^4+4c^2d(cd-e)e(cd+e)x\right)\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}\,dx}{2c^3e} - \cdots$$

$$= -\frac{abe\left(6c^2d^2-e^2\right)x}{2c^3} - \frac{bde^2x^2\left(a+b\tan^{-1}(cx)\right)}{c} - \frac{be^3x^3\left(a+b\tan^{-1}(cx)\right)}{6c} + \frac{(d+ex)\cdots}{}$$

$$= \frac{b^2de^2x}{c^2} - \frac{abe\left(6c^2d^2-e^2\right)x}{2c^3} - \frac{b^2e\left(6c^2d^2-e^2\right)x\tan^{-1}(cx)}{2c^3} - \frac{bde^2x^2\left(a+b\tan^{-1}(cx)\cdots\right.}{c}$$

$$= \frac{b^2de^2x}{c^2} - \frac{abe\left(6c^2d^2-e^2\right)x}{2c^3} - \frac{b^2de^2\tan^{-1}(cx)}{c^3} - \frac{b^2e\left(6c^2d^2-e^2\right)x\tan^{-1}(cx)}{2c^3} - \frac{bde^2\cdots}{}$$

$$= \frac{b^2de^2x}{c^2} - \frac{abe\left(6c^2d^2-e^2\right)x}{2c^3} + \frac{b^2e^3x^2}{12c^2} - \frac{b^2de^2\tan^{-1}(cx)}{c^3} - \frac{b^2e\left(6c^2d^2-e^2\right)x\tan^{-1}(c\cdots}{2c^3}$$

$$= \frac{b^2de^2x}{c^2} - \frac{abe\left(6c^2d^2-e^2\right)x}{2c^3} + \frac{b^2e^3x^2}{12c^2} - \frac{b^2de^2\tan^{-1}(cx)}{c^3} - \frac{b^2e\left(6c^2d^2-e^2\right)x\tan^{-1}(c\cdots}{2c^3}$$

$$= \frac{b^2de^2x}{c^2} - \frac{abe\left(6c^2d^2-e^2\right)x}{2c^3} + \frac{b^2e^3x^2}{12c^2} - \frac{b^2de^2\tan^{-1}(cx)}{c^3} - \frac{b^2e\left(6c^2d^2-e^2\right)x\tan^{-1}(c\cdots}{2c^3}$$

Mathematica [A] time = 0.946089, size = 472, normalized size = 1.26

$$\frac{-12ib^2cd\left(c^2d^2-e^2\right)\text{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right) + 18a^2c^4d^2ex^2 + 12a^2c^4d^3x + 12a^2c^4de^2x^3 + 3a^2c^4e^3x^4 + 2b\tan^{-1}(cx)\left(3a\cdots\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^3*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (b^2*e^3 + 12*a^2*c^4*d^3*x - 36*a*b*c^3*d^2*e*x + 12*b^2*c^2*d*e^2*x + 6*a
*b*c*e^3*x + 18*a^2*c^4*d^2*e*x^2 - 12*a*b*c^3*d*e^2*x^2 + b^2*c^2*e^3*x^2
+ 12*a^2*c^4*d*e^2*x^3 - 2*a*b*c^3*e^3*x^3 + 3*a^2*c^4*e^3*x^4 + 3*b^2*((-4
*I)*c^3*d^3 + 6*c^2*d^2*e + (4*I)*c*d*e^2 - e^3 + c^4*x*(4*d^3 + 6*d^2*e*x
+ 4*d*e^2*x^2 + e^3*x^3))*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*(-(b*c*e*(18*c^2*
```

```
d^2*x + e^2*x*(-3 + c^2*x^2) + 6*d*(e + c^2*e*x^2))) + 3*a*(6*c^2*d^2*e - e
^3 + c^4*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3)) + 12*b*c*d*(c^2*d^2
 - e^2)*Log[1 + E^((2*I)*ArcTan[c*x])]) - 12*a*b*c^3*d^3*Log[1 + c^2*x^2] +
 18*b^2*c^2*d^2*e*Log[1 + c^2*x^2] + 12*a*b*c*d*e^2*Log[1 + c^2*x^2] - 4*b^
2*e^3*Log[1 + c^2*x^2] - (12*I)*b^2*c*d*(c^2*d^2 - e^2)*PolyLog[2, -E^((2*I
)*ArcTan[c*x])])/(12*c^4)
```

Maple [B] time = 0.075, size = 948, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(a+b*arctan(c*x))^2,x)
```

```
[Out] -1/3*b^2*e^3*ln(c^2*x^2+1)/c^4+1/12*b^2*e^3*x^2/c^2+2*a*b*arctan(c*x)*x*d^3
+3/2*b^2*e*arctan(c*x)^2*x^2*d^2+b^2*e^2*arctan(c*x)^2*x^3*d+1/2*a*b*e^3*ar
ctan(c*x)*x^4-1/4*I/c*b^2*d^3*ln(c*x+I)^2-1/2*I/c*b^2*d^3*dilog(1/2*I*(c*x-
I))+1/4*I/c*b^2*d^3*ln(c*x-I)^2+1/2*I/c*b^2*d^3*dilog(-1/2*I*(c*x+I))-1/2/c
^4*a*b*e^3*arctan(c*x)+3/2/c^2*b^2*e*arctan(c*x)^2*d^2+3/2/c^2*b^2*e*ln(c^2
*x^2+1)*d^2+1/2/c^3*b^2*e^3*arctan(c*x)*x-1/6/c*b^2*e^3*arctan(c*x)*x^3-1/c
*b^2*arctan(c*x)*ln(c^2*x^2+1)*d^3-1/c*a*b*ln(c^2*x^2+1)*d^3-1/6/c*a*b*e^3*
x^3+1/2*a*b/c^3*e^3*x+1/4*a^2*e^3*x^4+a^2*x*d^3+3*a*b*e*arctan(c*x)*x^2*d^2
+2*a*b*e^2*arctan(c*x)*x^3*d+1/4*I/c^3*b^2*e^2*d*ln(c*x+I)^2-1/4*I/c^3*b^2*
e^2*d*ln(c*x-I)^2+1/2*I/c*b^2*d^3*ln(c*x-I)*ln(-1/2*I*(c*x+I))+b^2*d*e^2*x/
c^2-b^2*d*e^2*arctan(c*x)/c^3+1/2*I/c*b^2*d^3*ln(c^2*x^2+1)*ln(c*x+I)-1/2*I
/c*b^2*d^3*ln(c*x+I)*ln(1/2*I*(c*x-I))-1/2*I/c*b^2*d^3*ln(c^2*x^2+1)*ln(c*x
-I)+1/2*I/c^3*b^2*e^2*d*dilog(1/2*I*(c*x-I))-1/2*I/c^3*b^2*e^2*d*dilog(-1/2
*I*(c*x+I))-1/c*a*b*e^2*d*x^2-3*a*b/c*e*d^2*x-1/c*b^2*e^2*arctan(c*x)*d*x^2
-3/c*b^2*e*arctan(c*x)*d^2*x+3/c^2*a*b*e*arctan(c*x)*d^2+1/c^3*b^2*e^2*arct
an(c*x)*ln(c^2*x^2+1)*d+1/c^3*a*b*e^2*ln(c^2*x^2+1)*d+1/4*a^2/e*d^4+3/2*a^2
*e*x^2*d^2+a^2*e^2*x^3*d+b^2*arctan(c*x)^2*x*d^3+1/4*b^2*e^3*arctan(c*x)^2*
x^4-1/4/c^4*b^2*e^3*arctan(c*x)^2+1/2*I/c^3*b^2*e^2*d*ln(c^2*x^2+1)*ln(c*x-
I)-1/2*I/c^3*b^2*e^2*d*ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/2*I/c^3*b^2*e^2*d*ln(
c^2*x^2+1)*ln(c*x+I)+1/2*I/c^3*b^2*e^2*d*ln(c*x+I)*ln(1/2*I*(c*x-I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*e^3*x^4 + a^2*d*e^2*x^3 + 12*b^2*c^2*e^3*integrate(1/16*x^5*arctan(
c*x)^2/(c^2*x^2 + 1), x) + b^2*c^2*e^3*integrate(1/16*x^5*log(c^2*x^2 + 1)^
2/(c^2*x^2 + 1), x) + 36*b^2*c^2*d*e^2*integrate(1/16*x^4*arctan(c*x)^2/(c^
2*x^2 + 1), x) + b^2*c^2*e^3*integrate(1/16*x^5*log(c^2*x^2 + 1)/(c^2*x^2 +
 1), x) + 3*b^2*c^2*d*e^2*integrate(1/16*x^4*log(c^2*x^2 + 1)^2/(c^2*x^2 +
1), x) + 36*b^2*c^2*d^2*e*integrate(1/16*x^3*arctan(c*x)^2/(c^2*x^2 + 1), x
) + 4*b^2*c^2*d*e^2*integrate(1/16*x^4*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) +
 3*b^2*c^2*d^2*e*integrate(1/16*x^3*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) +
12*b^2*c^2*d^3*integrate(1/16*x^2*arctan(c*x)^2/(c^2*x^2 + 1), x) + 6*b^2*c
^2*d^2*e*integrate(1/16*x^3*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + b^2*c^2*d^
3*integrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 4*b^2*c^2*d^3*i
ntegrate(1/16*x^2*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 3/2*a^2*d^2*e*x^2 +
1/4*b^2*d^3*arctan(c*x)^3/c - 2*b^2*c*e^3*integrate(1/16*x^4*arctan(c*x)/(c
^2*x^2 + 1), x) - 8*b^2*c*d*e^2*integrate(1/16*x^3*arctan(c*x)/(c^2*x^2 + 1
), x) - 12*b^2*c*d^2*e*integrate(1/16*x^2*arctan(c*x)/(c^2*x^2 + 1), x) - 8
*b^2*c*d^3*integrate(1/16*x*arctan(c*x)/(c^2*x^2 + 1), x) + 3*(x^2*arctan(c
*x) - c*(x/c^2 - arctan(c*x)/c^3))*a*b*d^2*e + (2*x^3*arctan(c*x) - c*(x^2/
c^2 - log(c^2*x^2 + 1)/c^4))*a*b*d*e^2 + 1/6*(3*x^4*arctan(c*x) - c*((c^2*x
^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a*b*e^3 + a^2*d^3*x + 12*b^2*e^3*integr
ate(1/16*x^3*arctan(c*x)^2/(c^2*x^2 + 1), x) + b^2*e^3*integrate(1/16*x^3*l
og(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 36*b^2*d*e^2*integrate(1/16*x^2*arcta
n(c*x)^2/(c^2*x^2 + 1), x) + 3*b^2*d*e^2*integrate(1/16*x^2*log(c^2*x^2 + 1
)^2/(c^2*x^2 + 1), x) + 36*b^2*d^2*e*integrate(1/16*x*arctan(c*x)^2/(c^2*x^
2 + 1), x) + 3*b^2*d^2*e*integrate(1/16*x*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1),
 x) + b^2*d^3*integrate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + (2*c*x*
arctan(c*x) - log(c^2*x^2 + 1))*a*b*d^3/c + 1/16*(b^2*e^3*x^4 + 4*b^2*d*e^2
*x^3 + 6*b^2*d^2*e*x^2 + 4*b^2*d^3*x)*arctan(c*x)^2 - 1/64*(b^2*e^3*x^4 + 4
*b^2*d*e^2*x^3 + 6*b^2*d^2*e*x^2 + 4*b^2*d^3*x)*log(c^2*x^2 + 1)^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(a^2e^3x^3 + 3\,a^2de^2x^2 + 3\,a^2d^2ex + a^2d^3 + \left(b^2e^3x^3 + 3\,b^2de^2x^2 + 3\,b^2d^2ex + b^2d^3\right)\arctan\left(cx\right)^2 + 2\left(abe^3x^3 + 3\,abd\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*e^3*x^3 + 3*a^2*d*e^2*x^2 + 3*a^2*d^2*e*x + a^2*d^3 + (b^2*e^3
*x^3 + 3*b^2*d*e^2*x^2 + 3*b^2*d^2*e*x + b^2*d^3)*arctan(c*x)^2 + 2*(a*b*e^
```

```
3*x^3 + 3*a*b*d*e^2*x^2 + 3*a*b*d^2*e*x + a*b*d^3)*arctan(c*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{atan}(cx))^2 (d + ex)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(a+b*atan(c*x))**2,x)
```

```
[Out] Integral((a + b*atan(c*x))**2*(d + e*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^3 (b \arctan(cx) + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(b*arctan(c*x) + a)^2, x)
```

3.10 $\int (d+ex)^2 \left(a+b\tan^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=270

$$\frac{ib^2\left(3c^2d^2-e^2\right)\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^3}+\frac{i\left(3c^2d^2-e^2\right)\left(a+b\tan^{-1}(cx)\right)^2}{3c^3}-\frac{d\left(d^2-\frac{3e^2}{c^2}\right)\left(a+b\tan^{-1}(cx)\right)^2}{3e}+\frac{2b\left(3c^2d^2$$

```
[Out] (-2*a*b*d*e*x)/c + (b^2*e^2*x)/(3*c^2) - (b^2*e^2*ArcTan[c*x])/(3*c^3) - (2
*b^2*d*e*x*ArcTan[c*x])/c - (b*e^2*x^2*(a + b*ArcTan[c*x]))/(3*c) + ((I/3)*
(3*c^2*d^2 - e^2)*(a + b*ArcTan[c*x])^2)/c^3 - (d*(d^2 - (3*e^2)/c^2)*(a +
b*ArcTan[c*x])^2)/(3*e) + ((d + e*x)^3*(a + b*ArcTan[c*x])^2)/(3*e) + (2*b*
(3*c^2*d^2 - e^2)*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(3*c^3) + (b^2*d*
e*Log[1 + c^2*x^2])/c^2 + ((I/3)*b^2*(3*c^2*d^2 - e^2)*PolyLog[2, 1 - 2/(1
+ I*c*x)])/c^3
```

Rubi [A] time = 0.398677, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4864, 4846, 260, 4852, 321, 203, 4984, 4884, 4920, 4854, 2402, 2315}

$$\frac{ib^2\left(3c^2d^2-e^2\right)\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^3}+\frac{i\left(3c^2d^2-e^2\right)\left(a+b\tan^{-1}(cx)\right)^2}{3c^3}-\frac{d\left(d^2-\frac{3e^2}{c^2}\right)\left(a+b\tan^{-1}(cx)\right)^2}{3e}+\frac{2b\left(3c^2d^2$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (-2*a*b*d*e*x)/c + (b^2*e^2*x)/(3*c^2) - (b^2*e^2*ArcTan[c*x])/(3*c^3) - (2
*b^2*d*e*x*ArcTan[c*x])/c - (b*e^2*x^2*(a + b*ArcTan[c*x]))/(3*c) + ((I/3)*
(3*c^2*d^2 - e^2)*(a + b*ArcTan[c*x])^2)/c^3 - (d*(d^2 - (3*e^2)/c^2)*(a +
b*ArcTan[c*x])^2)/(3*e) + ((d + e*x)^3*(a + b*ArcTan[c*x])^2)/(3*e) + (2*b*
(3*c^2*d^2 - e^2)*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(3*c^3) + (b^2*d*
e*Log[1 + c^2*x^2])/c^2 + ((I/3)*b^2*(3*c^2*d^2 - e^2)*PolyLog[2, 1 - 2/(1
+ I*c*x)])/c^3
```

Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
```

```
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4984

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int (d+ex)^2 \left(a+b\tan^{-1}(cx)\right)^2 dx = \frac{(d+ex)^3\left(a+b\tan^{-1}(cx)\right)^2}{3e} - \frac{(2bc)\int \left(\frac{3de^2\left(a+b\tan^{-1}(cx)\right)}{c^2} + \frac{e^3x\left(a+b\tan^{-1}(cx)\right)}{c^2} + \frac{(c^2d^3-3 \cdots}{\cdots}\right.}{3e}$$

$$= \frac{(d+ex)^3\left(a+b\tan^{-1}(cx)\right)^2}{3e} - \frac{(2b)\int \frac{\left(c^2d^3-3de^2+e\left(3c^2d^2-e^2\right)x\right)\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}dx}{3ce} - \frac{(2bde \cdots}{\cdots}$$

$$= -\frac{2abdex}{c} - \frac{be^2x^2\left(a+b\tan^{-1}(cx)\right)}{3c} + \frac{(d+ex)^3\left(a+b\tan^{-1}(cx)\right)^2}{3e} - \frac{(2b)\int \left(\frac{c^2d^3\left(1- \cdots\right.}{\cdots}\right.}{\cdots}$$

$$= -\frac{2abdex}{c} + \frac{b^2e^2x}{3c^2} - \frac{2b^2dex\tan^{-1}(cx)}{c} - \frac{be^2x^2\left(a+b\tan^{-1}(cx)\right)}{3c} + \frac{(d+ex)^3\left(a+b \cdots\right.}{3e}$$

$$= -\frac{2abdex}{c} + \frac{b^2e^2x}{3c^2} - \frac{b^2e^2\tan^{-1}(cx)}{3c^3} - \frac{2b^2dex\tan^{-1}(cx)}{c} - \frac{be^2x^2\left(a+b\tan^{-1}(cx)\right)}{3c} \cdots$$

$$= -\frac{2abdex}{c} + \frac{b^2e^2x}{3c^2} - \frac{b^2e^2\tan^{-1}(cx)}{3c^3} - \frac{2b^2dex\tan^{-1}(cx)}{c} - \frac{be^2x^2\left(a+b\tan^{-1}(cx)\right)}{3c} \cdots$$

$$= -\frac{2abdex}{c} + \frac{b^2e^2x}{3c^2} - \frac{b^2e^2\tan^{-1}(cx)}{3c^3} - \frac{2b^2dex\tan^{-1}(cx)}{c} - \frac{be^2x^2\left(a+b\tan^{-1}(cx)\right)}{3c} \cdots$$

$$= -\frac{2abdex}{c} + \frac{b^2e^2x}{3c^2} - \frac{b^2e^2\tan^{-1}(cx)}{3c^3} - \frac{2b^2dex\tan^{-1}(cx)}{c} - \frac{be^2x^2\left(a+b\tan^{-1}(cx)\right)}{3c} \cdots$$

<u>Mathematica</u> [A] time = 0.573217, size = 312, normalized size = 1.16

$$\frac{-ib^2\left(3c^2d^2-e^2\right)\mathrm{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right) + 3a^2c^3d^2x + 3a^2c^3dex^2 + a^2c^3e^2x^3 + b\tan^{-1}(cx)\left(2ac^3x\left(3d^2+3dex+e^2x^2 \cdots\right.\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^2*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (3*a^2*c^3*d^2*x - 6*a*b*c^2*d*e*x + b^2*c*e^2*x + 3*a^2*c^3*d*e*x^2 - a*b*
c^2*e^2*x^2 + a^2*c^3*e^2*x^3 + b^2*((-3*I)*c^2*d^2 + 3*c*d*e + I*e^2 + c^3
*x*(3*d^2 + 3*d*e*x + e^2*x^2))*ArcTan[c*x]^2 + b*ArcTan[c*x]*(6*a*c*d*e -
b*e*(e + 6*c^2*d*x + c^2*e*x^2) + 2*a*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2) + 2
*b*(3*c^2*d^2 - e^2)*Log[1 + E^((2*I)*ArcTan[c*x])]) - 3*a*b*c^2*d^2*Log[1
+ c^2*x^2] + 3*b^2*c*d*e*Log[1 + c^2*x^2] + a*b*e^2*Log[1 + c^2*x^2] - I*b^
```

```
2*(3*c^2*d^2 - e^2)*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(3*c^3)
```

Maple [B] time = 0.069, size = 750, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(a+b*arctan(c*x))^2,x)
```

```
[Out] -1/6*I/c^3*b^2*dilog(-1/2*I*(c*x+I))*e^2+1/3*b^2*e^2*x/c^2-1/3*b^2*e^2*arct
an(c*x)/c^3-1/c*a*b*ln(c^2*x^2+1)*d^2-1/3/c*b^2*e^2*arctan(c*x)*x^2-1/c*b^2
*arctan(c*x)*ln(c^2*x^2+1)*d^2+1/c^2*b^2*e*arctan(c*x)^2*d+1/3/c^3*b^2*e^2*
arctan(c*x)*ln(c^2*x^2+1)+1/3/c^3*a*b*e^2*ln(c^2*x^2+1)+b^2*e*arctan(c*x)^2
*x^2*d+2/3*a*b*e^2*arctan(c*x)*x^3+2*a*b*arctan(c*x)*x*d^2-1/4*I/c*b^2*ln(c
*x+I)^2*d^2-1/2*I/c*b^2*dilog(1/2*I*(c*x-I))*d^2+1/4*I/c*b^2*ln(c*x-I)^2*d^
2+1/2*I/c*b^2*dilog(-1/2*I*(c*x+I))*d^2+1/6*I/c^3*b^2*dilog(1/2*I*(c*x-I))*
e^2+1/12*I/c^3*b^2*ln(c*x+I)^2*e^2-1/12*I/c^3*b^2*ln(c*x-I)^2*e^2-1/3/c*a*b
*x^2*e^2+a^2*x*d^2+1/3*a^2*e^2*x^3+b^2*d*e*ln(c^2*x^2+1)/c^2+1/2*I/c*b^2*ln
(c^2*x^2+1)*ln(c*x+I)*d^2+1/6*I/c^3*b^2*ln(c*x+I)*ln(1/2*I*(c*x-I))*e^2+2/c
^2*a*b*e*arctan(c*x)*d-1/2*I/c*b^2*ln(c*x+I)*ln(1/2*I*(c*x-I))*d^2-1/2*I/c*
b^2*ln(c^2*x^2+1)*ln(c*x-I)*d^2+1/2*I/c*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))*d^
2+2*a*b*e*arctan(c*x)*x^2*d+1/6*I/c^3*b^2*ln(c^2*x^2+1)*ln(c*x-I)*e^2-1/6*I
/c^3*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))*e^2-1/6*I/c^3*b^2*ln(c^2*x^2+1)*ln(c*
x+I)*e^2+1/3*a^2/e*d^3+a^2*e*x^2*d+b^2*arctan(c*x)^2*x*d^2+1/3*b^2*e^2*arct
an(c*x)^2*x^3-2*a*b*d*e*x/c-2*b^2*d*e*x*arctan(c*x)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*e^2*x^3 + 36*b^2*c^2*e^2*integrate(1/48*x^4*arctan(c*x)^2/(c^2*x^2
+ 1), x) + 3*b^2*c^2*e^2*integrate(1/48*x^4*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1
), x) + 72*b^2*c^2*d*e*integrate(1/48*x^3*arctan(c*x)^2/(c^2*x^2 + 1), x) +
 4*b^2*c^2*e^2*integrate(1/48*x^4*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 6*b^
```

```
2*c^2*d*e*integrate(1/48*x^3*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 36*b^2*
c^2*d^2*integrate(1/48*x^2*arctan(c*x)^2/(c^2*x^2 + 1), x) + 12*b^2*c^2*d*e
*integrate(1/48*x^3*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 3*b^2*c^2*d^2*inte
grate(1/48*x^2*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 12*b^2*c^2*d^2*integr
ate(1/48*x^2*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + a^2*d*e*x^2 + 1/4*b^2*d^2
*arctan(c*x)^3/c - 8*b^2*c*e^2*integrate(1/48*x^3*arctan(c*x)/(c^2*x^2 + 1)
, x) - 24*b^2*c*d*e*integrate(1/48*x^2*arctan(c*x)/(c^2*x^2 + 1), x) - 24*b
^2*c*d^2*integrate(1/48*x*arctan(c*x)/(c^2*x^2 + 1), x) + 2*(x^2*arctan(c*x
) - c*(x/c^2 - arctan(c*x)/c^3))*a*b*d*e + 1/3*(2*x^3*arctan(c*x) - c*(x^2/
c^2 - log(c^2*x^2 + 1)/c^4))*a*b*e^2 + a^2*d^2*x + 36*b^2*e^2*integrate(1/4
8*x^2*arctan(c*x)^2/(c^2*x^2 + 1), x) + 3*b^2*e^2*integrate(1/48*x^2*log(c^
2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 72*b^2*d*e*integrate(1/48*x*arctan(c*x)^2/
(c^2*x^2 + 1), x) + 6*b^2*d*e*integrate(1/48*x*log(c^2*x^2 + 1)^2/(c^2*x^2
+ 1), x) + 3*b^2*d^2*integrate(1/48*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) +
(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a*b*d^2/c + 1/12*(b^2*e^2*x^3 + 3*b^
2*d*e*x^2 + 3*b^2*d^2*x)*arctan(c*x)^2 - 1/48*(b^2*e^2*x^3 + 3*b^2*d*e*x^2
+ 3*b^2*d^2*x)*log(c^2*x^2 + 1)^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(a^2e^2x^2 + 2\,a^2dex + a^2d^2 + \left(b^2e^2x^2 + 2\,b^2dex + b^2d^2\right)\arctan\left(cx\right)^2 + 2\left(abe^2x^2 + 2\,abdex + abd^2\right)\arctan\left(cx\right),\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*e^2*x^2 + 2*a^2*d*e*x + a^2*d^2 + (b^2*e^2*x^2 + 2*b^2*d*e*x +
 b^2*d^2)*arctan(c*x)^2 + 2*(a*b*e^2*x^2 + 2*a*b*d*e*x + a*b*d^2)*arctan(c*
x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\,\text{atan}\left(cx\right)\right)^2\left(d + ex\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(a+b*atan(c*x))**2,x)
```

```
[Out] Integral((a + b*atan(c*x))**2*(d + e*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^2 (b \arctan(cx) + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(b*arctan(c*x) + a)^2, x)
```

3.11 $\int (d+ex)\left(a+b\tan^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=171

$$\frac{ib^2 d\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c}-\frac{\left(d^2-\frac{e^2}{c^2}\right)\left(a+b\tan^{-1}(cx)\right)^2}{2e}+\frac{(d+ex)^2\left(a+b\tan^{-1}(cx)\right)^2}{2e}+\frac{id\left(a+b\tan^{-1}(cx)\right)^2}{c}+\frac{2b}{}$$

```
[Out] -((a*b*e*x)/c) - (b^2*e*x*ArcTan[c*x])/c + (I*d*(a + b*ArcTan[c*x])^2)/c -
((d^2 - e^2/c^2)*(a + b*ArcTan[c*x])^2)/(2*e) + ((d + e*x)^2*(a + b*ArcTan[
c*x])^2)/(2*e) + (2*b*d*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c + (b^2*e*
Log[1 + c^2*x^2])/(2*c^2) + (I*b^2*d*PolyLog[2, 1 - 2/(1 + I*c*x)])/c
```

Rubi [A] time = 0.297722, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {4864, 4846, 260, 4984, 4884, 4920, 4854, 2402, 2315}

$$\frac{ib^2 d\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c}-\frac{\left(d^2-\frac{e^2}{c^2}\right)\left(a+b\tan^{-1}(cx)\right)^2}{2e}+\frac{(d+ex)^2\left(a+b\tan^{-1}(cx)\right)^2}{2e}+\frac{id\left(a+b\tan^{-1}(cx)\right)^2}{c}+\frac{2b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] -((a*b*e*x)/c) - (b^2*e*x*ArcTan[c*x])/c + (I*d*(a + b*ArcTan[c*x])^2)/c -
((d^2 - e^2/c^2)*(a + b*ArcTan[c*x])^2)/(2*e) + ((d + e*x)^2*(a + b*ArcTan[
c*x])^2)/(2*e) + (2*b*d*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c + (b^2*e*
Log[1 + c^2*x^2])/(2*c^2) + (I*b^2*d*PolyLog[2, 1 - 2/(1 + I*c*x)])/c
```

Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
```

```
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4984

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int (d+ex)\left(a+b\tan^{-1}(cx)\right)^2\,dx = \frac{(d+ex)^2\left(a+b\tan^{-1}(cx)\right)^2}{2e} - \frac{(bc)\int\left(\frac{e^2\left(a+b\tan^{-1}(cx)\right)}{c^2}+\frac{\left(c^2d^2-e^2+2c^2dex\right)\left(a+b\tan^{-1}(cx)\right)}{c^2\left(1+c^2x^2\right)}\right)}{e}$$

$$= \frac{(d+ex)^2\left(a+b\tan^{-1}(cx)\right)^2}{2e} - \frac{b\int\frac{\left(c^2d^2-e^2+2c^2dex\right)\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}\,dx}{ce} - \frac{(be)\int\left(a+b\tan\right.}{c}$$

$$= -\frac{abex}{c} + \frac{(d+ex)^2\left(a+b\tan^{-1}(cx)\right)^2}{2e} - \frac{b\int\left(\frac{c^2d^2\left(1-\frac{e^2}{c^2d^2}\right)\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}+\frac{2c^2dex\left(a+b\tan\right.}{1+c^2x^2}\right.}{ce}$$

$$= -\frac{abex}{c} - \frac{b^2ex\tan^{-1}(cx)}{c} + \frac{(d+ex)^2\left(a+b\tan^{-1}(cx)\right)^2}{2e} - (2bcd)\int\frac{x\left(a+b\tan^{-1}(c\right.}{1+c^2x^2}$$

$$= -\frac{abex}{c} - \frac{b^2ex\tan^{-1}(cx)}{c} + \frac{id\left(a+b\tan^{-1}(cx)\right)^2}{c} - \frac{\left(d^2-\frac{e^2}{c^2}\right)\left(a+b\tan^{-1}(cx)\right)^2}{2e} + \frac{(a}{}$$

$$= -\frac{abex}{c} - \frac{b^2ex\tan^{-1}(cx)}{c} + \frac{id\left(a+b\tan^{-1}(cx)\right)^2}{c} - \frac{\left(d^2-\frac{e^2}{c^2}\right)\left(a+b\tan^{-1}(cx)\right)^2}{2e} + \frac{(a}{}$$

$$= -\frac{abex}{c} - \frac{b^2ex\tan^{-1}(cx)}{c} + \frac{id\left(a+b\tan^{-1}(cx)\right)^2}{c} - \frac{\left(d^2-\frac{e^2}{c^2}\right)\left(a+b\tan^{-1}(cx)\right)^2}{2e} + \frac{(a}{}$$

$$= -\frac{abex}{c} - \frac{b^2ex\tan^{-1}(cx)}{c} + \frac{id\left(a+b\tan^{-1}(cx)\right)^2}{c} - \frac{\left(d^2-\frac{e^2}{c^2}\right)\left(a+b\tan^{-1}(cx)\right)^2}{2e} + \frac{(a}{}$$

Mathematica [A] time = 0.269229, size = 172, normalized size = 1.01

$$\frac{-2ib^2cd\,\text{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)+2a^2c^2dx+a^2c^2ex^2+2b\tan^{-1}(cx)\left(a\left(2c^2dx+c^2ex^2+e\right)+2bcd\log\left(1+e^{2i\tan^{-1}(cx}\right.\right.}{2c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (2*a^2*c^2*d*x - 2*a*b*c*e*x + a^2*c^2*e*x^2 + b^2*(-I + c*x)*(2*c*d + I*e
+ c*e*x)*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*(-(b*c*e*x) + a*(e + 2*c^2*d*x + c
^2*e*x^2) + 2*b*c*d*Log[1 + E^((2*I)*ArcTan[c*x])]) - 2*a*b*c*d*Log[1 + c^2
*x^2] + b^2*e*Log[1 + c^2*x^2] - (2*I)*b^2*c*d*PolyLog[2, -E^((2*I)*ArcTan[
c*x])])/(2*c^2)
```

Maple [B] time = 0.091, size = 360, normalized size = 2.1

$$\frac{a^2x^2e}{2}+a^2dx+\frac{b^2\left(\arctan\left(cx\right)\right)^2x^2e}{2}+b^2\left(\arctan\left(cx\right)\right)^2xd-\frac{b^2\arctan\left(cx\right)\ln\left(c^2x^2+1\right)d}{c}+\frac{b^2\left(\arctan\left(cx\right)\right)^2e}{2c^2}-\frac{b^2ex}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(a+b*arctan(c*x))^2,x)
```

```
[Out] 1/2*a^2*x^2*e+a^2*d*x+1/2*b^2*arctan(c*x)^2*x^2*e+b^2*arctan(c*x)^2*x*d-1/c
*b^2*arctan(c*x)*ln(c^2*x^2+1)*d+1/2/c^2*b^2*arctan(c*x)^2*e-b^2*e*x*arctan
(c*x)/c+1/2*b^2*e*ln(c^2*x^2+1)/c^2+1/2*I/c*b^2*d*ln(c*x-I)*ln(-1/2*I*(c*x+
I))+1/2*I/c*b^2*d*ln(c^2*x^2+1)*ln(c*x+I)-1/2*I/c*b^2*d*dilog(1/2*I*(c*x-I)
)-1/4*I/c*b^2*d*ln(c*x+I)^2-1/2*I/c*b^2*d*ln(c^2*x^2+1)*ln(c*x-I)+1/2*I/c*b
^2*d*dilog(-1/2*I*(c*x+I))-1/2*I/c*b^2*d*ln(c*x+I)*ln(1/2*I*(c*x-I))+1/4*I/
c*b^2*d*ln(c*x-I)^2+a*b*arctan(c*x)*x^2*e+2*a*b*arctan(c*x)*x*d-a*b*e*x/c-1
/c*a*b*d*ln(c^2*x^2+1)+1/c^2*a*b*e*arctan(c*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$12\,b^2c^2e\int\frac{x^3\arctan\left(cx\right)^2}{16\left(c^2x^2+1\right)}\,dx+b^2c^2e\int\frac{x^3\log\left(c^2x^2+1\right)^2}{16\left(c^2x^2+1\right)}\,dx+12\,b^2c^2d\int\frac{x^2\arctan\left(cx\right)^2}{16\left(c^2x^2+1\right)}\,dx+2\,b^2c^2e\int\frac{x^3\log\left(c^2x^2\right.}{16\left(c^2x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 12*b^2*c^2*e*integrate(1/16*x^3*arctan(c*x)^2/(c^2*x^2 + 1), x) + b^2*c^2*e
*integrate(1/16*x^3*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 12*b^2*c^2*d*int
egrate(1/16*x^2*arctan(c*x)^2/(c^2*x^2 + 1), x) + 2*b^2*c^2*e*integrate(1/1
6*x^3*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + b^2*c^2*d*integrate(1/16*x^2*log
(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 4*b^2*c^2*d*integrate(1/16*x^2*log(c^2*
x^2 + 1)/(c^2*x^2 + 1), x) + 1/2*a^2*e*x^2 + 1/4*b^2*d*arctan(c*x)^3/c - 4*
b^2*c*e*integrate(1/16*x^2*arctan(c*x)/(c^2*x^2 + 1), x) - 8*b^2*c*d*integr
ate(1/16*x*arctan(c*x)/(c^2*x^2 + 1), x) + (x^2*arctan(c*x) - c*(x/c^2 - ar
ctan(c*x)/c^3))*a*b*e + a^2*d*x + 12*b^2*e*integrate(1/16*x*arctan(c*x)^2/(
c^2*x^2 + 1), x) + b^2*e*integrate(1/16*x*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1),
 x) + b^2*d*integrate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + (2*c*x*ar
ctan(c*x) - log(c^2*x^2 + 1))*a*b*d/c + 1/8*(b^2*e*x^2 + 2*b^2*d*x)*arctan(
```

```
c*x)^2 - 1/32*(b^2*e*x^2 + 2*b^2*d*x)*log(c^2*x^2 + 1)^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(a^2ex + a^2d + \left(b^2ex + b^2d\right)\arctan(cx)^2 + 2\,(abex + abd)\arctan(cx)\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*e*x + a^2*d + (b^2*e*x + b^2*d)*arctan(c*x)^2 + 2*(a*b*e*x + a
*b*d)*arctan(c*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\,\text{atan}(cx))^2\,(d + ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*atan(c*x))**2,x)
```

```
[Out] Integral((a + b*atan(c*x))**2*(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex + d)(b\arctan(cx) + a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(b*arctan(c*x) + a)^2, x)
```

3.12 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d+ex}\,dx$

Optimal. Leaf size=223

$$-\frac{ib\left(a+b\tan^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e}+\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)}{e}+\frac{b^2\text{PolyLog}\left(3,1-\frac{}{(1-}\right.}{2e}$$

```
[Out] -(((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e) + ((a + b*ArcTan[c*x])^2*Lo
g[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e + (I*b*(a + b*ArcTan[c*x])*
PolyLog[2, 1 - 2/(1 - I*c*x)])/e - (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 -
(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e - (b^2*PolyLog[3, 1 - 2/(1 -
I*c*x)])/(2*e) + (b^2*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*
x))])/(2*e)
```

Rubi [A] time = 0.0487754, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {4858}

$$-\frac{ib\left(a+b\tan^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e}+\frac{ib\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)}{e}+\frac{b^2\text{PolyLog}\left(3,1-\frac{}{(1-}\right.}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(d + e*x),x]
```

```
[Out] -(((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e) + ((a + b*ArcTan[c*x])^2*Lo
g[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e + (I*b*(a + b*ArcTan[c*x])*
PolyLog[2, 1 - 2/(1 - I*c*x)])/e - (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 -
(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e - (b^2*PolyLog[3, 1 - 2/(1 -
I*c*x)])/(2*e) + (b^2*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*
x))])/(2*e)
```

Rule 4858

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
-Simp[((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e, x] + (Simp[((a + b*ArcT
an[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x] + Simp[(I*
b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/e, x] - Simp[(I*b*(a +
 b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/
e, x] - Simp[(b^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e), x] + Simp[(b^2*Poly
```

```
Log[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e), x]) /; FreeQ[
{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d+ex}\,dx = -\frac{\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1-icx}\right)}{e} + \frac{\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e} + \frac{ib\left(a+b\tan^{-1}(cx\right.}{e}$$

Mathematica [F] time = 114.81, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{d+ex}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(d + e*x),x]
```

```
[Out] Integrate[(a + b*ArcTan[c*x])^2/(d + e*x), x]
```

Maple [C] time = 0.911, size = 1297, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/(e*x+d),x)
```

```
[Out] a^2*ln(c*e*x+c*d)/e+b^2*ln(c*e*x+c*d)/e*arctan(c*x)^2-b^2/e*arctan(c*x)^2*l
n(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+d*c)-I*a*b*l
n(c*e*x+c*d)/e*ln((I*e+e*c*x)/(I*e-d*c))-1/2*I*b^2/e*arctan(c*x)^2*Pi*csgn(
I*(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+d*c)/((1+I*
c*x)^2/(c^2*x^2+1)+1))^2*csgn(I*(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)
^2/(c^2*x^2+1)+I*e+d*c))-1/2*I*b^2/e*arctan(c*x)^2*Pi*csgn(I/((1+I*c*x)^2/(
c^2*x^2+1)+1))*csgn(I*(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)^2/(c^2*x^
2+1)+I*e+d*c)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2+1/2*I*b^2/e*arctan(c*x)^2*Pi*c
sgn(I*(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+d*c)/((
1+I*c*x)^2/(c^2*x^2+1)+1))^3+I*a*b*ln(c*e*x+c*d)/e*ln((I*e-e*c*x)/(d*c+I*e)
)-1/2*b^2/e*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+c*b^2/e*d/(d*c-I*e)*arctan(
```

```
c*x)^2*ln(1-(I*e-d*c)/(d*c+I*e)*(1+I*c*x)^2/(c^2*x^2+1))-I*c*b^2/e*d/(d*c-I
*e)*arctan(c*x)*polylog(2,(I*e-d*c)/(d*c+I*e)*(1+I*c*x)^2/(c^2*x^2+1))+1/2*
c*b^2/e*d/(d*c-I*e)*polylog(3,(I*e-d*c)/(d*c+I*e)*(1+I*c*x)^2/(c^2*x^2+1))+
b^2*arctan(c*x)^2*ln(1-(I*e-d*c)/(d*c+I*e)*(1+I*c*x)^2/(c^2*x^2+1))/(e+I*d*
c)+I*b^2/e*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+1/2*b^2*polylog(
3,(I*e-d*c)/(d*c+I*e)*(1+I*c*x)^2/(c^2*x^2+1))/(e+I*d*c)+2*a*b*ln(c*e*x+c*d
)/e*arctan(c*x)-I*a*b/e*dilog((I*e+e*c*x)/(I*e-d*c))+1/2*I*b^2/e*arctan(c*x
)^2*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*(-I*(1+I*c*x)^2/(c^2*x^2+
1)*e+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+d*c)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn
(I*(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+d*c))+I*a*
b/e*dilog((I*e-e*c*x)/(d*c+I*e))-I*b^2*arctan(c*x)*polylog(2,(I*e-d*c)/(d*c
+I*e)*(1+I*c*x)^2/(c^2*x^2+1))/(e+I*d*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2 \log(ex+d)}{e} + \int \frac{12\,b^2 \arctan(cx)^2 + b^2 \log\left(c^2x^2+1\right)^2 + 32\,ab \arctan(cx)}{16\,(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] a^2*log(e*x + d)/e + integrate(1/16*(12*b^2*arctan(c*x)^2 + b^2*log(c^2*x^2
 + 1)^2 + 32*a*b*arctan(c*x))/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \arctan(cx)^2 + 2\,ab \arctan(cx) + a^2}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{atan}(cx))^2}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/(e*x+d),x)
```

```
[Out] Integral((a + b*atan(c*x))**2/(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(cx) + a)^2}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2/(e*x + d), x)
```

3.13 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{(d+ex)^2}\,dx$

Optimal. Leaf size=341

$$\frac{ib^2c\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{c^2d^2+e^2}+\frac{ib^2c\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^2d^2+e^2}-\frac{ib^2c\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{c^2d^2+e^2}+\frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{c^2d^2+e^2}+\frac{c}{}$$

```
[Out] (I*c*(a + b*ArcTan[c*x])^2)/(c^2*d^2 + e^2) + (c^2*d*(a + b*ArcTan[c*x])^2)
/(e*(c^2*d^2 + e^2)) - (a + b*ArcTan[c*x])^2/(e*(d + e*x)) - (2*b*c*(a + b*
ArcTan[c*x])*Log[2/(1 - I*c*x)])/(c^2*d^2 + e^2) + (2*b*c*(a + b*ArcTan[c*x
])*Log[2/(1 + I*c*x)])/(c^2*d^2 + e^2) + (2*b*c*(a + b*ArcTan[c*x])*Log[(2*
c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(c^2*d^2 + e^2) + (I*b^2*c*PolyLog
[2, 1 - 2/(1 - I*c*x)])/(c^2*d^2 + e^2) + (I*b^2*c*PolyLog[2, 1 - 2/(1 + I*
c*x)])/(c^2*d^2 + e^2) - (I*b^2*c*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*
e)*(1 - I*c*x))])/(c^2*d^2 + e^2)
```

Rubi [A] time = 0.3701, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {4864, 4856, 2402, 2315, 2447, 4984, 4884, 4920, 4854}

$$\frac{ib^2c\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{c^2d^2+e^2}+\frac{ib^2c\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^2d^2+e^2}-\frac{ib^2c\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{c^2d^2+e^2}+\frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{c^2d^2+e^2}+\frac{c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(d + e*x)^2,x]
```

```
[Out] (I*c*(a + b*ArcTan[c*x])^2)/(c^2*d^2 + e^2) + (c^2*d*(a + b*ArcTan[c*x])^2)
/(e*(c^2*d^2 + e^2)) - (a + b*ArcTan[c*x])^2/(e*(d + e*x)) - (2*b*c*(a + b*
ArcTan[c*x])*Log[2/(1 - I*c*x)])/(c^2*d^2 + e^2) + (2*b*c*(a + b*ArcTan[c*x
])*Log[2/(1 + I*c*x)])/(c^2*d^2 + e^2) + (2*b*c*(a + b*ArcTan[c*x])*Log[(2*
c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(c^2*d^2 + e^2) + (I*b^2*c*PolyLog
[2, 1 - 2/(1 - I*c*x)])/(c^2*d^2 + e^2) + (I*b^2*c*PolyLog[2, 1 - 2/(1 + I*
c*x)])/(c^2*d^2 + e^2) - (I*b^2*c*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*
e)*(1 - I*c*x))])/(c^2*d^2 + e^2)
```

Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4984

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
```

```
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{(d+ex)^2}\,dx = -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{e(d+ex)} + \frac{(2bc)\int \left(\frac{e^2\left(a+b\tan^{-1}(cx)\right)}{\left(c^2d^2+e^2\right)(d+ex)} + \frac{c^2(d-ex)\left(a+b\tan^{-1}(cx)\right)}{\left(c^2d^2+e^2\right)\left(1+c^2x^2\right)}\right)dx}{e}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{e(d+ex)} + \frac{\left(2bc^3\right)\int \frac{(d-ex)\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}\,dx}{e\left(c^2d^2+e^2\right)} + \frac{(2bce)\int \frac{a+b\tan^{-1}(cx)}{d+ex}\,dx}{c^2d^2+e^2}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{e(d+ex)} - \frac{2bc\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{c^2d^2+e^2} + \frac{2bc\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2c(d+\cdots}{(cd+ie)(\cdots}\right.}{c^2d^2+e^2}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{e(d+ex)} - \frac{2bc\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{c^2d^2+e^2} + \frac{2bc\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2c(d+\cdots}{(cd+ie)(\cdots}\right.}{c^2d^2+e^2}$$

$$= \frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{c^2d^2+e^2} + \frac{c^2d\left(a+b\tan^{-1}(cx)\right)^2}{e\left(c^2d^2+e^2\right)} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{e(d+ex)} - \frac{2bc\left(a+b\tan^{-1}(cx)\right)\cdots}{c^2d^2+e^2}$$

$$= \frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{c^2d^2+e^2} + \frac{c^2d\left(a+b\tan^{-1}(cx)\right)^2}{e\left(c^2d^2+e^2\right)} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{e(d+ex)} - \frac{2bc\left(a+b\tan^{-1}(cx)\right)\cdots}{c^2d^2+e^2}$$

$$= \frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{c^2d^2+e^2} + \frac{c^2d\left(a+b\tan^{-1}(cx)\right)^2}{e\left(c^2d^2+e^2\right)} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{e(d+ex)} - \frac{2bc\left(a+b\tan^{-1}(cx)\right)\cdots}{c^2d^2+e^2}$$

$$= \frac{ic\left(a+b\tan^{-1}(cx)\right)^2}{c^2d^2+e^2} + \frac{c^2d\left(a+b\tan^{-1}(cx)\right)^2}{e\left(c^2d^2+e^2\right)} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{e(d+ex)} - \frac{2bc\left(a+b\tan^{-1}(cx)\right)\cdots}{c^2d^2+e^2}$$

Mathematica [A] time = 2.88078, size = 300, normalized size = 0.88

$$\frac{b^2\left(-\frac{cd\left(i\,\mathrm{PolyLog}\left(2,e^{2i\left(\tan^{-1}\left(\frac{cd}{e}\right)+\tan^{-1}(cx)\right)}\right)-\frac{1}{2}\pi\log\left(c^2x^2+1\right)-i\tan^{-1}(cx)\left(\pi-2\tan^{-1}\left(\frac{cd}{e}\right)\right)-2\left(\tan^{-1}\left(\frac{cd}{e}\right)+\tan^{-1}(cx)\right)\log\left(1-e^{2i\left(\tan^{-1}\left(\frac{cd}{e}\right)+\tan^{-1}(cx)\right)}\right)+\cdots\right)}{c^2d^2+e^2}\cdots\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(d + e*x)^2,x]
```

```
[Out] -(a^2/(e*(d + e*x))) + (a*b*(-2*(e - c^2*d*x)*ArcTan[c*x] + c*(d + e*x)*(2*
Log[c*(d + e*x)] - Log[1 + c^2*x^2])))/((c^2*d^2 + e^2)*(d + e*x)) + (b^2*(
```

```
-((E^(I*ArcTan[(c*d)/e])*ArcTan[c*x]^2)/(Sqrt[1 + (c^2*d^2)/e^2]*e)) + (x*A
rcTan[c*x]^2)/(d + e*x) - (c*d*((-I)*(Pi - 2*ArcTan[(c*d)/e])*ArcTan[c*x] -
 Pi*Log[1 + E^((-2*I)*ArcTan[c*x])] - 2*(ArcTan[(c*d)/e] + ArcTan[c*x])*Log
[1 - E^((2*I)*(ArcTan[(c*d)/e] + ArcTan[c*x]))] - (Pi*Log[1 + c^2*x^2])/2 +
 2*ArcTan[(c*d)/e]*Log[Sin[ArcTan[(c*d)/e] + ArcTan[c*x]]] + I*PolyLog[2, E
^((2*I)*(ArcTan[(c*d)/e] + ArcTan[c*x]))]))/(c^2*d^2 + e^2)))/d
```

Maple [B] time = 0.102, size = 698, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/(e*x+d)^2,x)
```

```
[Out] -c*a^2/(c*e*x+c*d)/e-c*b^2/(c*e*x+c*d)/e*arctan(c*x)^2+2*c*b^2*arctan(c*x)/
(c^2*d^2+e^2)*ln(c*e*x+c*d)-c*b^2*arctan(c*x)/(c^2*d^2+e^2)*ln(c^2*x^2+1)+c
^2*b^2/e/(c^2*d^2+e^2)*d*arctan(c*x)^2+I*c*b^2/(c^2*d^2+e^2)*ln(c*e*x+c*d)*
ln((I*e-e*c*x)/(d*c+I*e))-1/2*I*c*b^2/(c^2*d^2+e^2)*ln(c*x-I)*ln(c^2*x^2+1)
+1/4*I*c*b^2/(c^2*d^2+e^2)*ln(c*x-I)^2-I*c*b^2/(c^2*d^2+e^2)*ln(c*e*x+c*d)*
ln((I*e+e*c*x)/(I*e-d*c))-1/2*I*c*b^2/(c^2*d^2+e^2)*dilog(1/2*I*(c*x-I))-I*
c*b^2/(c^2*d^2+e^2)*dilog((I*e+e*c*x)/(I*e-d*c))-1/4*I*c*b^2/(c^2*d^2+e^2)*
ln(c*x+I)^2-1/2*I*c*b^2/(c^2*d^2+e^2)*ln(c*x+I)*ln(1/2*I*(c*x-I))+1/2*I*c*b
^2/(c^2*d^2+e^2)*dilog(-1/2*I*(c*x+I))+I*c*b^2/(c^2*d^2+e^2)*dilog((I*e-e*c
*x)/(d*c+I*e))+1/2*I*c*b^2/(c^2*d^2+e^2)*ln(c*x+I)*ln(c^2*x^2+1)+1/2*I*c*b^
2/(c^2*d^2+e^2)*ln(c*x-I)*ln(-1/2*I*(c*x+I))-2*c*a*b/(c*e*x+c*d)/e*arctan(c
*x)+2*c*a*b/(c^2*d^2+e^2)*ln(c*e*x+c*d)-c*a*b/(c^2*d^2+e^2)*ln(c^2*x^2+1)+2
*c^2*a*b/e/(c^2*d^2+e^2)*d*arctan(c*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(\left(\frac{2\,cd\arctan\left(cx\right)}{c^2d^2e+e^3}-\frac{\log\left(c^2x^2+1\right)}{c^2d^2+e^2}+\frac{2\,\log\left(ex+d\right)}{c^2d^2+e^2}\right)c-\frac{2\,\arctan\left(cx\right)}{e^2x+de}\right)ab-\frac{\frac{1}{4}\left(28\,\arctan\left(cx\right)^2-4\left(e^2x+de\right)\int\frac{36\left(c^2ex^2\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] ((2*c*d*arctan(c*x)/(c^2*d^2*e + e^3) - log(c^2*x^2 + 1)/(c^2*d^2 + e^2) +
2*log(e*x + d)/(c^2*d^2 + e^2))*c - 2*arctan(c*x)/(e^2*x + d*e))*a*b - 1/16
*(4*arctan(c*x)^2 - 16*(e^2*x + d*e)*integrate(1/16*(12*(c^2*e*x^2 + e)*arc
tan(c*x)^2 + (c^2*e*x^2 + e)*log(c^2*x^2 + 1)^2 + 8*(c*e*x + c*d)*arctan(c*
x) - 4*(c^2*e*x^2 + c^2*d*x)*log(c^2*x^2 + 1))/(c^2*e^3*x^4 + 2*c^2*d*e^2*x
^3 + 2*d*e^2*x + d^2*e + (c^2*d^2*e + e^3)*x^2), x) - log(c^2*x^2 + 1)^2)*b
^2/(e^2*x + d*e) - a^2/(e^2*x + d*e)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \arctan(cx)^2 + 2\,ab \arctan(cx) + a^2}{e^2x^2 + 2\,dex + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(e^2*x^2 + 2*d*e*x +
 d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{atan}(cx))^2}{(d + ex)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/(e*x+d)**2,x)
```

```
[Out] Integral((a + b*atan(c*x))**2/(d + e*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(cx) + a)^2}{(ex + d)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(e*x+d)^2,x, algorithm="giac")

[Out] integrate((b*arctan(c*x) + a)^2/(e*x + d)^2, x)
```

3.14 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{(d+ex)^3}\,dx$

Optimal. Leaf size=496

$$\frac{ib^2c^3d\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{\left(c^2d^2+e^2\right)^2}+\frac{ib^2c^3d\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{\left(c^2d^2+e^2\right)^2}-\frac{ib^2c^3d\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{\left(c^2d^2+e^2\right)^2}+\frac{ic^3d\left(a+b\tan^{-1}\right.}{\left(c^2d^2+e^2\right)}$$

```
[Out] (b^2*c^3*d*ArcTan[c*x])/(c^2*d^2 + e^2)^2 - (b*c*(a + b*ArcTan[c*x]))/((c^2
*d^2 + e^2)*(d + e*x)) + (I*c^3*d*(a + b*ArcTan[c*x])^2)/(c^2*d^2 + e^2)^2
+ (c^2*(c*d - e)*(c*d + e)*(a + b*ArcTan[c*x])^2)/(2*e*(c^2*d^2 + e^2)^2) -
 (a + b*ArcTan[c*x])^2/(2*e*(d + e*x)^2) - (2*b*c^3*d*(a + b*ArcTan[c*x])*L
og[2/(1 - I*c*x)])/(c^2*d^2 + e^2)^2 + (2*b*c^3*d*(a + b*ArcTan[c*x])*Log[2
/(1 + I*c*x)])/(c^2*d^2 + e^2)^2 + (b^2*c^2*e*Log[d + e*x])/(c^2*d^2 + e^2)
^2 + (2*b*c^3*d*(a + b*ArcTan[c*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I
*c*x))])/(c^2*d^2 + e^2)^2 - (b^2*c^2*e*Log[1 + c^2*x^2])/(2*(c^2*d^2 + e^2
)^2) + (I*b^2*c^3*d*PolyLog[2, 1 - 2/(1 - I*c*x)])/(c^2*d^2 + e^2)^2 + (I*b
^2*c^3*d*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^2*d^2 + e^2)^2 - (I*b^2*c^3*d*Po
lyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(c^2*d^2 + e^2)^2
```

Rubi [A] time = 0.538568, antiderivative size = 496, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 15, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {4864, 4862, 706, 31, 635, 203, 260, 4856, 2402, 2315, 2447, 4984, 4884, 4920, 4854}

$$\frac{ib^2c^3d\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{\left(c^2d^2+e^2\right)^2}+\frac{ib^2c^3d\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{\left(c^2d^2+e^2\right)^2}-\frac{ib^2c^3d\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{\left(c^2d^2+e^2\right)^2}+\frac{ic^3d\left(a+b\tan^{-1}\right.}{\left(c^2d^2+e^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/(d + e*x)^3,x]
```

```
[Out] (b^2*c^3*d*ArcTan[c*x])/(c^2*d^2 + e^2)^2 - (b*c*(a + b*ArcTan[c*x]))/((c^2
*d^2 + e^2)*(d + e*x)) + (I*c^3*d*(a + b*ArcTan[c*x])^2)/(c^2*d^2 + e^2)^2
+ (c^2*(c*d - e)*(c*d + e)*(a + b*ArcTan[c*x])^2)/(2*e*(c^2*d^2 + e^2)^2) -
 (a + b*ArcTan[c*x])^2/(2*e*(d + e*x)^2) - (2*b*c^3*d*(a + b*ArcTan[c*x])*L
og[2/(1 - I*c*x)])/(c^2*d^2 + e^2)^2 + (2*b*c^3*d*(a + b*ArcTan[c*x])*Log[2
/(1 + I*c*x)])/(c^2*d^2 + e^2)^2 + (b^2*c^2*e*Log[d + e*x])/(c^2*d^2 + e^2)
^2 + (2*b*c^3*d*(a + b*ArcTan[c*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I
```

```
*c*x))])/(c^2*d^2 + e^2)^2 - (b^2*c^2*e*Log[1 + c^2*x^2])/(2*(c^2*d^2 + e^2
)^2) + (I*b^2*c^3*d*PolyLog[2, 1 - 2/(1 - I*c*x)])/(c^2*d^2 + e^2)^2 + (I*b
^2*c^3*d*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^2*d^2 + e^2)^2 - (I*b^2*c^3*d*Po
lyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(c^2*d^2 + e^2)^2
```

Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4984

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{(d+ex)^3}\,dx = -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2e(d+ex)^2} + \frac{(bc)\int\left(\frac{e^2\left(a+b\tan^{-1}(cx)\right)}{\left(c^2d^2+e^2\right)(d+ex)^2} + \frac{2c^2de^2\left(a+b\tan^{-1}(cx)\right)}{\left(c^2d^2+e^2\right)^2(d+ex)} + \frac{\left(c^4d^2-c^2e^2-2c^4dex\right)\left(a+b\tan^{-1}\right.}{\left(c^2d^2+e^2\right)^2\left(1+c^2x^2\right)}\right.}{e}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2e(d+ex)^2} + \frac{(bc)\int\frac{\left(c^4d^2-c^2e^2-2c^4dex\right)\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}\,dx}{e\left(c^2d^2+e^2\right)^2} + \frac{\left(2bc^3de\right)\int\frac{a+b\tan^{-1}(cx)}{d+ex}\,dx}{\left(c^2d^2+e^2\right)^2} +$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{\left(c^2d^2+e^2\right)(d+ex)} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2e(d+ex)^2} - \frac{2bc^3d\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{\left(c^2d^2+e^2\right)^2} + \frac{2bc^3d\,(\ldots}{}$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{\left(c^2d^2+e^2\right)(d+ex)} - \frac{\left(a+b\tan^{-1}(cx)\right)^2}{2e(d+ex)^2} - \frac{2bc^3d\left(a+b\tan^{-1}(cx)\right)\log\left(\frac{2}{1-icx}\right)}{\left(c^2d^2+e^2\right)^2} + \frac{2bc^3d\,(\ldots}{}$$

$$= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{\left(c^2d^2+e^2\right)(d+ex)} + \frac{ic^3d\left(a+b\tan^{-1}(cx)\right)^2}{\left(c^2d^2+e^2\right)^2} + \frac{c^2(cd-e)(cd+e)\left(a+b\tan^{-1}(cx)\right)^2}{2e\left(c^2d^2+e^2\right)^2} - \frac{(\ldots}{}$$

$$= \frac{b^2c^3d\tan^{-1}(cx)}{\left(c^2d^2+e^2\right)^2} - \frac{bc\left(a+b\tan^{-1}(cx)\right)}{\left(c^2d^2+e^2\right)(d+ex)} + \frac{ic^3d\left(a+b\tan^{-1}(cx)\right)^2}{\left(c^2d^2+e^2\right)^2} + \frac{c^2(cd-e)(cd+e)\left(a+b\ldots\right.}{2e\left(c^2d^2+e^2\ldots\right.}$$

$$= \frac{b^2c^3d\tan^{-1}(cx)}{\left(c^2d^2+e^2\right)^2} - \frac{bc\left(a+b\tan^{-1}(cx)\right)}{\left(c^2d^2+e^2\right)(d+ex)} + \frac{ic^3d\left(a+b\tan^{-1}(cx)\right)^2}{\left(c^2d^2+e^2\right)^2} + \frac{c^2(cd-e)(cd+e)\left(a+b\ldots\right.}{2e\left(c^2d^2+e^2\ldots\right.}$$

$$= \frac{b^2c^3d\tan^{-1}(cx)}{\left(c^2d^2+e^2\right)^2} - \frac{bc\left(a+b\tan^{-1}(cx)\right)}{\left(c^2d^2+e^2\right)(d+ex)} + \frac{ic^3d\left(a+b\tan^{-1}(cx)\right)^2}{\left(c^2d^2+e^2\right)^2} + \frac{c^2(cd-e)(cd+e)\left(a+b\ldots\right.}{2e\left(c^2d^2+e^2\ldots\right.}$$

<u>Mathematica</u> [A] time = 6.03814, size = 479, normalized size = 0.97

$$\frac{b^2c^2\left(-\frac{2cd\left(i\,\mathrm{PolyLog}\left(2,e^{2i\left(\tan^{-1}\left(\frac{cd}{e}\right)+\tan^{-1}(cx)\right)}\right)-\frac{1}{2}\pi\log\left(c^2x^2+1\right)-i\tan^{-1}(cx)\left(\pi-2\tan^{-1}\left(\frac{cd}{e}\right)\right)-2\left(\tan^{-1}\left(\frac{cd}{e}\right)+\tan^{-1}(cx)\right)\log\left(1-e^{2i\left(\tan^{-1}\left(\frac{cd}{e}\right)+\tan^{-1}(cx)\right.}\right.\right.}{c^2d^2+e^2}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/(d + e*x)^3,x]
```

```
[Out] -a^2/(2*e*(d + e*x)^2) + (a*b*((-e^3 + c^4*d^2*x*(2*d + e*x) - c^2*e*(3*d^2
 + 2*d*e*x + e^2*x^2))*ArcTan[c*x] + c*(d + e*x)*(-(c^2*d^2) - e^2 + 2*c^2*
d*(d + e*x)*Log[c*(d + e*x)] - c^2*d*(d + e*x)*Log[1 + c^2*x^2])))/((c^2*d^
2 + e^2)^2*(d + e*x)^2) + (b^2*c^2*((-2*E^(I*ArcTan[(c*d)/e])*ArcTan[c*x]^2
)/(Sqrt[1 + (c^2*d^2)/e^2]*e) - (e*(1 + c^2*x^2)*ArcTan[c*x]^2)/(c^2*(d + e
*x)^2) + (2*x*ArcTan[c*x]*(e + c*d*ArcTan[c*x]))/(c*d*(d + e*x)) + (-2*e^2*
ArcTan[c*x] + 2*c*d*e*Log[(c*(d + e*x))/Sqrt[1 + c^2*x^2]])/(c^3*d^3 + c*d*
e^2) - (2*c*d*((-I)*(Pi - 2*ArcTan[(c*d)/e])*ArcTan[c*x] - Pi*Log[1 + E^((-
2*I)*ArcTan[c*x])] - 2*(ArcTan[(c*d)/e] + ArcTan[c*x])*Log[1 - E^((2*I)*(Ar
cTan[(c*d)/e] + ArcTan[c*x]))] - (Pi*Log[1 + c^2*x^2])/2 + 2*ArcTan[(c*d)/e
]*Log[Sin[ArcTan[(c*d)/e] + ArcTan[c*x]]] + I*PolyLog[2, E^((2*I)*(ArcTan[(
c*d)/e] + ArcTan[c*x]))]))/(c^2*d^2 + e^2)))/(2*(c^2*d^2 + e^2))
```

Maple [B] time = 0.102, size = 961, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/(e*x+d)^3,x)
```

```
[Out] -c^2*a*b/(c^2*d^2+e^2)/(c*e*x+c*d)-1/2*b^2*c^2*e*ln(c^2*x^2+1)/(c^2*d^2+e^2
)^2-1/2*c^2*b^2/(c*e*x+c*d)^2/e*arctan(c*x)^2-c^2*b^2*arctan(c*x)/(c^2*d^2+
e^2)/(c*e*x+c*d)+c^2*b^2*e/(c^2*d^2+e^2)^2*ln(c*e*x+c*d)-1/2*c^2*b^2*e/(c^2
*d^2+e^2)^2*arctan(c*x)^2+b^2*c^3*d*arctan(c*x)/(c^2*d^2+e^2)^2-1/2*c^2*a^2
/(c*e*x+c*d)^2/e-I*c^3*b^2*d/(c^2*d^2+e^2)^2*ln(c*e*x+c*d)*ln((I*e+e*c*x)/(
I*e-d*c))+1/2*I*c^3*b^2*d/(c^2*d^2+e^2)^2*ln(c*x+I)*ln(c^2*x^2+1)+1/2*I*c^3
*b^2*d/(c^2*d^2+e^2)^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/2*I*c^3*b^2*d/(c^2*d^
2+e^2)^2*ln(c*x-I)*ln(c^2*x^2+1)-1/2*I*c^3*b^2*d/(c^2*d^2+e^2)^2*ln(c*x+I)*
ln(1/2*I*(c*x-I))+I*c^3*b^2*d/(c^2*d^2+e^2)^2*ln(c*e*x+c*d)*ln((I*e-e*c*x)/
(d*c+I*e))+c^4*a*b/e/(c^2*d^2+e^2)^2*arctan(c*x)*d^2-c^2*a*b*e/(c^2*d^2+e^2
)^2*arctan(c*x)+2*c^3*a*b*d/(c^2*d^2+e^2)^2*ln(c*e*x+c*d)+I*c^3*b^2*d/(c^2*
d^2+e^2)^2*dilog((I*e-e*c*x)/(d*c+I*e))+1/2*c^4*b^2/e/(c^2*d^2+e^2)^2*arcta
n(c*x)^2*d^2-I*c^3*b^2*d/(c^2*d^2+e^2)^2*dilog((I*e+e*c*x)/(I*e-d*c))-1/4*I
*c^3*b^2*d/(c^2*d^2+e^2)^2*ln(c*x+I)^2+1/2*I*c^3*b^2*d/(c^2*d^2+e^2)^2*dilo
g(-1/2*I*(c*x+I))+1/4*I*c^3*b^2*d/(c^2*d^2+e^2)^2*ln(c*x-I)^2-1/2*I*c^3*b^2
*d/(c^2*d^2+e^2)^2*dilog(1/2*I*(c*x-I))+2*c^3*b^2*arctan(c*x)*d/(c^2*d^2+e^
2)^2*ln(c*e*x+c*d)-c^3*b^2*arctan(c*x)/(c^2*d^2+e^2)^2*d*ln(c^2*x^2+1)-c^2*
a*b/(c*e*x+c*d)^2/e*arctan(c*x)-c^3*a*b/(c^2*d^2+e^2)^2*d*ln(c^2*x^2+1)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \arctan(cx)^2 + 2ab\arctan(cx) + a^2}{e^3x^3 + 3de^2x^2 + 3d^2ex + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(e^3*x^3 + 3*d*e^2*x
^2 + 3*d^2*e*x + d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx) + a)^2}{(ex + d)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^2/(e*x + d)^3, x)
```

3.15 $\int (d+ex)^3 \left(a+b\tan^{-1}(cx)\right)^3 dx$

Optimal. Leaf size=652

$$\frac{3ib^2d(cd-e)(cd+e)\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^3} - \frac{3ib^3e\left(6c^2d^2-e^2\right)\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{4c^4} + \frac{3b^3d(cd-e$$

```
[Out] (3*a*b^2*d*e^2*x)/c^2 - (b^3*e^3*x)/(4*c^3) + (b^3*e^3*ArcTan[c*x])/(4*c^4)
 + (3*b^3*d*e^2*x*ArcTan[c*x])/c^2 + (b^2*e^3*x^2*(a + b*ArcTan[c*x]))/(4*c
^2) - (3*b*d*e^2*(a + b*ArcTan[c*x])^2)/(2*c^3) + ((I/4)*b*e^3*(a + b*ArcTa
n[c*x])^2)/c^4 - (((3*I)/4)*b*e*(6*c^2*d^2 - e^2)*(a + b*ArcTan[c*x])^2)/c^
4 - (3*b*e*(6*c^2*d^2 - e^2)*x*(a + b*ArcTan[c*x])^2)/(4*c^3) - (3*b*d*e^2*
x^2*(a + b*ArcTan[c*x])^2)/(2*c) - (b*e^3*x^3*(a + b*ArcTan[c*x])^2)/(4*c)
+ (I*d*(c*d - e)*(c*d + e)*(a + b*ArcTan[c*x])^3)/c^3 - ((c^4*d^4 - 6*c^2*d
^2*e^2 + e^4)*(a + b*ArcTan[c*x])^3)/(4*c^4*e) + ((d + e*x)^4*(a + b*ArcTan
[c*x])^3)/(4*e) + (b^2*e^3*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(2*c^4)
- (3*b^2*e*(6*c^2*d^2 - e^2)*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(2*c^4
) + (3*b*d*(c*d - e)*(c*d + e)*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/c^
3 - (3*b^3*d*e^2*Log[1 + c^2*x^2])/(2*c^3) + ((I/4)*b^3*e^3*PolyLog[2, 1 -
2/(1 + I*c*x)])/c^4 - (((3*I)/4)*b^3*e*(6*c^2*d^2 - e^2)*PolyLog[2, 1 - 2/(
1 + I*c*x)])/c^4 + ((3*I)*b^2*d*(c*d - e)*(c*d + e)*(a + b*ArcTan[c*x])*Pol
yLog[2, 1 - 2/(1 + I*c*x)])/c^3 + (3*b^3*d*(c*d - e)*(c*d + e)*PolyLog[3, 1
 - 2/(1 + I*c*x)])/(2*c^3)
```

Rubi [A] time = 1.20167, antiderivative size = 652, normalized size of antiderivative = 1., number of steps used = 29, number of rules used = 15, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {4864, 4846, 4920, 4854, 2402, 2315, 4852, 4916, 260, 4884, 321, 203, 4984, 4994, 6610}

$$\frac{3ib^2d(cd-e)(cd+e)\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^3} - \frac{3ib^3e\left(6c^2d^2-e^2\right)\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{4c^4} + \frac{3b^3d(cd-e$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (3*a*b^2*d*e^2*x)/c^2 - (b^3*e^3*x)/(4*c^3) + (b^3*e^3*ArcTan[c*x])/(4*c^4)
 + (3*b^3*d*e^2*x*ArcTan[c*x])/c^2 + (b^2*e^3*x^2*(a + b*ArcTan[c*x]))/(4*c
^2) - (3*b*d*e^2*(a + b*ArcTan[c*x])^2)/(2*c^3) + ((I/4)*b*e^3*(a + b*ArcTa
n[c*x])^2)/c^4 - (((3*I)/4)*b*e*(6*c^2*d^2 - e^2)*(a + b*ArcTan[c*x])^2)/c^
```

```
4 - (3*b*e*(6*c^2*d^2 - e^2)*x*(a + b*ArcTan[c*x])^2)/(4*c^3) - (3*b*d*e^2*
x^2*(a + b*ArcTan[c*x])^2)/(2*c) - (b*e^3*x^3*(a + b*ArcTan[c*x])^2)/(4*c)
 + (I*d*(c*d - e)*(c*d + e)*(a + b*ArcTan[c*x])^3)/c^3 - ((c^4*d^4 - 6*c^2*d
^2*e^2 + e^4)*(a + b*ArcTan[c*x])^3)/(4*c^4*e) + ((d + e*x)^4*(a + b*ArcTan
[c*x])^3)/(4*e) + (b^2*e^3*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(2*c^4)
 - (3*b^2*e*(6*c^2*d^2 - e^2)*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(2*c^4
) + (3*b*d*(c*d - e)*(c*d + e)*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/c^
3 - (3*b^3*d*e^2*Log[1 + c^2*x^2])/(2*c^3) + ((I/4)*b^3*e^3*PolyLog[2, 1 -
 2/(1 + I*c*x)])/c^4 - (((3*I)/4)*b^3*e*(6*c^2*d^2 - e^2)*PolyLog[2, 1 - 2/(
1 + I*c*x)])/c^4 + ((3*I)*b^2*d*(c*d - e)*(c*d + e)*(a + b*ArcTan[c*x])*Pol
yLog[2, 1 - 2/(1 + I*c*x)])/c^3 + (3*b^3*d*(c*d - e)*(c*d + e)*PolyLog[3, 1
 - 2/(1 + I*c*x)])/(2*c^3)
```

Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 4984

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int (d+ex)^3\left(a+b\tan^{-1}(cx)\right)^3\,dx = \frac{(d+ex)^4\left(a+b\tan^{-1}(cx)\right)^3}{4e} - \frac{(3bc)\int\left(\frac{e^2\left(6c^2d^2-e^2\right)\left(a+b\tan^{-1}(cx)\right)^2}{c^4} + \frac{4de^3x\left(a+b\tan^{-1}(cx)\right)}{c^2}\right.}{}$$

$$= \frac{(d+ex)^4\left(a+b\tan^{-1}(cx)\right)^3}{4e} - \frac{(3b)\int \frac{\left(c^4d^4-6c^2d^2e^2+e^4+4c^2d(cd-e)e(cd+e)x\right)\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}\,d}{4c^3e}$$

$$= -\frac{3be\left(6c^2d^2-e^2\right)x\left(a+b\tan^{-1}(cx)\right)^2}{4c^3} - \frac{3bde^2x^2\left(a+b\tan^{-1}(cx)\right)^2}{2c} - \frac{be^3x^3\left(a+b\right.}{4}$$

$$= -\frac{3ibe\left(6c^2d^2-e^2\right)\left(a+b\tan^{-1}(cx)\right)^2}{4c^4} - \frac{3be\left(6c^2d^2-e^2\right)x\left(a+b\tan^{-1}(cx)\right)^2}{4c^3} - \frac{3b}{}$$

$$= \frac{3ab^2de^2x}{c^2} + \frac{b^2e^3x^2\left(a+b\tan^{-1}(cx)\right)}{4c^2} - \frac{3bde^2\left(a+b\tan^{-1}(cx)\right)^2}{2c^3} + \frac{ibe^3\left(a+b\tan\right.}{4c^4}$$

$$= \frac{3ab^2de^2x}{c^2} - \frac{b^3e^3x}{4c^3} + \frac{3b^3de^2x\tan^{-1}(cx)}{c^2} + \frac{b^2e^3x^2\left(a+b\tan^{-1}(cx)\right)}{4c^2} - \frac{3bde^2\left(a+b\right.}{2c}$$

$$= \frac{3ab^2de^2x}{c^2} - \frac{b^3e^3x}{4c^3} + \frac{b^3e^3\tan^{-1}(cx)}{4c^4} + \frac{3b^3de^2x\tan^{-1}(cx)}{c^2} + \frac{b^2e^3x^2\left(a+b\tan^{-1}(cx\right.}{4c^2}$$

$$= \frac{3ab^2de^2x}{c^2} - \frac{b^3e^3x}{4c^3} + \frac{b^3e^3\tan^{-1}(cx)}{4c^4} + \frac{3b^3de^2x\tan^{-1}(cx)}{c^2} + \frac{b^2e^3x^2\left(a+b\tan^{-1}(cx\right.}{4c^2}$$

Mathematica [A] time = 1.84167, size = 855, normalized size = 1.31

$$\frac{a^3e^3x^4c^4 + 3a^2bx\left(4d^3+6exd^2+4e^2x^2d+e^3x^3\right)\tan^{-1}(cx)c^4 + a^2e^2(4acd-be)x^3c^3 + 6a^2de(acd-be)x^2c^3 + 12ab^2d^3\left(\text{ta}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^3*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (a^2*c*(4*a*c^3*d^3 + 3*b*e*(-6*c^2*d^2 + e^2))*x + 6*a^2*c^3*d*e*(a*c*d -
b*e)*x^2 + a^2*c^3*e^2*(4*a*c*d - b*e)*x^3 + a^3*c^4*e^3*x^4 + 3*a^2*b*(6*c
^2*d^2*e - e^3)*ArcTan[c*x] + 3*a^2*b*c^4*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^
2 + e^3*x^3)*ArcTan[c*x] + a*b^2*e^3*(1 + c^2*x^2 + (6*c*x - 2*c^3*x^3)*Arc
Tan[c*x] + 3*(-1 + c^4*x^4)*ArcTan[c*x]^2 - 4*Log[1 + c^2*x^2]) - 6*a^2*b*c
*d*(c^2*d^2 - e^2)*Log[1 + c^2*x^2] + 18*a*b^2*c^2*d^2*e*(-2*c*x*ArcTan[c*x
```

```
] + (1 + c^2*x^2)*ArcTan[c*x]^2 + Log[1 + c^2*x^2]) + 12*a*b^2*c^3*d^3*(Arc
Tan[c*x]*((-I + c*x)*ArcTan[c*x] + 2*Log[1 + E^((2*I)*ArcTan[c*x])]) - I*Po
lyLog[2, -E^((2*I)*ArcTan[c*x])]) + 12*a*b^2*c*d*e^2*(c*x + (I + c^3*x^3)*A
rcTan[c*x]^2 - ArcTan[c*x]*(1 + c^2*x^2 + 2*Log[1 + E^((2*I)*ArcTan[c*x])])
 + I*PolyLog[2, -E^((2*I)*ArcTan[c*x])]) + 6*b^3*c^2*d^2*e*(ArcTan[c*x]*((3
*I - 3*c*x)*ArcTan[c*x] + (1 + c^2*x^2)*ArcTan[c*x]^2 - 6*Log[1 + E^((2*I)*
ArcTan[c*x])]) + (3*I)*PolyLog[2, -E^((2*I)*ArcTan[c*x])]) + b^3*e^3*(-(c*x
) - (4*I - 3*c*x + c^3*x^3)*ArcTan[c*x]^2 + (-1 + c^4*x^4)*ArcTan[c*x]^3 +
ArcTan[c*x]*(1 + c^2*x^2 + 8*Log[1 + E^((2*I)*ArcTan[c*x])]) - (4*I)*PolyLo
g[2, -E^((2*I)*ArcTan[c*x])]) + 2*b^3*c*d*e^2*(6*c*x*ArcTan[c*x] - 3*ArcTan
[c*x]^2 - 3*c^2*x^2*ArcTan[c*x]^2 + (2*I)*ArcTan[c*x]^3 + 2*c^3*x^3*ArcTan[
c*x]^3 - 6*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] - 3*Log[1 + c^2*x^2
] + (6*I)*ArcTan[c*x]*PolyLog[2, -E^((2*I)*ArcTan[c*x])] - 3*PolyLog[3, -E^
((2*I)*ArcTan[c*x])]) + 2*b^3*c^3*d^3*(2*ArcTan[c*x]^2*((-I + c*x)*ArcTan[c
*x] + 3*Log[1 + E^((2*I)*ArcTan[c*x])]) - (6*I)*ArcTan[c*x]*PolyLog[2, -E^(
(2*I)*ArcTan[c*x])] + 3*PolyLog[3, -E^((2*I)*ArcTan[c*x])]))/(4*c^4)
```

Maple [C] time = 3.813, size = 3577, normalized size = 5.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(a+b*arctan(c*x))^3,x)
```

```
[Out] 3/2*I/c^3*a*b^2*e^2*d*ln(c^2*x^2+1)*ln(c*x-I)+3/2*I/c^3*a*b^2*e^2*d*ln(c*x+
I)*ln(1/2*I*(c*x-I))-3/4*I/c^3*b^3*e^2*Pi*d*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)
+1)^2)^3*arctan(c*x)^2+3/4*I/c^3*b^3*e^2*Pi*d*csgn(I*(1+I*c*x)^2/(c^2*x^2+1
))^3*arctan(c*x)^2+3/4*I/c^3*b^3*e^2*Pi*d*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((
1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*arctan(c*x)^2+3/4*I/c^3*b^3*e^2*Pi*d*csgn(I/
((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*
c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*arctan(c*x)^2+3*a*b^2*d*e
^2*x/c^2+3*b^3*d*e^2*x*arctan(c*x)/c^2-1/4*b^3*e^3*x/c^3+1/4*b^3*e^3*arctan
(c*x)/c^4+1/4*a^3*e^3*x^4+a^3*x*d^3+3/4*I/c*b^3*Pi*d^3*csgn(I/((1+I*c*x)^2/
(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1
)^2)^2*arctan(c*x)^2-3/2*I/c*b^3*Pi*d^3*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))
*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*arctan(c*x)^2+3/2*I/c*b^3*Pi*d^3*c
sgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2*arctan
(c*x)^2-3/4*I/c*b^3*Pi*d^3*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2*csgn(I*(1+
I*c*x)^2/(c^2*x^2+1))*arctan(c*x)^2+3/4*I/c*b^3*Pi*d^3*csgn(I*(1+I*c*x)^2/(
c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2
*arctan(c*x)^2+3/4*I/c*b^3*Pi*d^3*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csg
```

```
n(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*arctan(c*x)^2-3/2*I/c^3*a*b^2*e^2*d*ln(c
^2*x^2+1)*ln(c*x+I)-3/2*I/c^3*a*b^2*e^2*d*ln(c*x-I)*ln(-1/2*I*(c*x+I))+a^3*
e^2*x^3*d+3/2*a^3*e*x^2*d^2+1/4*b^3*e^3*arctan(c*x)^3*x^4+b^3*arctan(c*x)^3
*x*d^3-1/4/c^4*b^3*e^3*arctan(c*x)^3+3/2/c*b^3*d^3*polylog(3,-(1+I*c*x)^2/(
c^2*x^2+1))+1/4*a^3/e*d^4-1/4*I/c^4*b^3*e^3-3/4*I/c^3*b^3*e^2*Pi*d*csgn(I/(
(1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(
c^2*x^2+1)+1)^2)^2*arctan(c*x)^2-3/4*I/c^3*b^3*e^2*Pi*d*csgn(I*((1+I*c*x)^2
/(c^2*x^2+1)+1))^2*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*arctan(c*x)^2-3/2*
I/c^3*b^3*e^2*Pi*d*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*csgn(I*(1+I*c*x)^2/(
c^2*x^2+1))^2*arctan(c*x)^2+3/4*I/c^3*b^3*e^2*Pi*d*csgn(I*(1+I*c*x)/(c^2*x^
2+1)^(1/2))^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*arctan(c*x)^2-3/4*I/c^3*b^3*e
^2*Pi*d*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+
I*c*x)^2/(c^2*x^2+1)+1)^2)^2*arctan(c*x)^2+3/2*I/c^3*b^3*e^2*Pi*d*csgn(I*((
1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*arctan(c
*x)^2-3/4*I/c*b^3*Pi*d^3*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*
c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)
+1)^2)*arctan(c*x)^2+3/4*a*b^2*e^3*arctan(c*x)^2*x^4+3*a^2*b*arctan(c*x)*x*
d^3+3*a*b^2*arctan(c*x)^2*x*d^3+3/4*a^2*b*e^3*arctan(c*x)*x^4+b^3*e^2*arcta
n(c*x)^3*x^3*d-1/4/c*b^3*e^3*arctan(c*x)^2*x^3+3/c*b^3*d^3*ln((1+I*c*x)/(c^
2*x^2+1)^(1/2))*arctan(c*x)^2-3/2/c*b^3*arctan(c*x)^2*ln(c^2*x^2+1)*d^3-3/4
/c^4*a^2*b*arctan(c*x)*e^3-1/c^4*a*b^2*e^3*ln(c^2*x^2+1)-3/4/c^4*a*b^2*e^3*
arctan(c*x)^2-3/2/c^3*b^3*e^2*d*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+3/c^3*b
^3*e^2*d*ln((1+I*c*x)^2/(c^2*x^2+1)+1)+1/4/c^2*b^3*e^3*arctan(c*x)*x^2+3/2/
c^2*b^3*e*arctan(c*x)^3*d^2-3/2/c^3*b^3*e^2*d*arctan(c*x)^2+3/c*b^3*ln(2)*d
^3*arctan(c*x)^2-3/2/c*a^2*b*ln(c^2*x^2+1)*d^3+2/c^4*b^3*e^3*arctan(c*x)*ln
(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+2/c^4*b^3*e^3*arctan(c*x)*ln(1-I*(1+I*c*x
)/(c^2*x^2+1)^(1/2))+3/4/c^3*b^3*e^3*arctan(c*x)^2*x+3/2*b^3*e*arctan(c*x)^
3*x^2*d^2-2*I/c^4*b^3*e^3*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-I/c^4*b^3*
e^3*arctan(c*x)^2-2*I/c^4*b^3*e^3*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-I/
c*b^3*d^3*arctan(c*x)^3-1/4/c*a^2*b*e^3*x^3+1/4/c^2*a*b^2*e^3*x^2+3/4*a^2*b
/c^3*e^3*x-3/c*a*b^2*e^2*arctan(c*x)*d*x^2-9/c*a*b^2*e*arctan(c*x)*d^2*x+3/
c^3*a*b^2*e^2*arctan(c*x)*ln(c^2*x^2+1)*d-3/2*I/c*a*b^2*d^3*ln(c^2*x^2+1)*l
n(c*x-I)-3/2*I/c*a*b^2*d^3*ln(c*x+I)*ln(1/2*I*(c*x-I))+3/2*I/c*a*b^2*d^3*ln
(c*x-I)*ln(-1/2*I*(c*x+I))+3/2*I/c*a*b^2*d^3*ln(c^2*x^2+1)*ln(c*x+I)-3/4*I/
c*b^3*Pi*d^3*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^
3*arctan(c*x)^2-3/4*I/c*b^3*Pi*d^3*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3*arctan
(c*x)^2+3/4*I/c*b^3*Pi*d^3*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*arctan(c
*x)^2-3/4*I/c^3*a*b^2*e^2*d*ln(c*x-I)^2+3/4*I/c^3*a*b^2*e^2*d*ln(c*x+I)^2+3
/2*I/c^3*a*b^2*e^2*d*dilog(1/2*I*(c*x-I))-3/2*I/c^3*a*b^2*e^2*d*dilog(-1/2*
I*(c*x+I))+3*I/c^3*b^3*e^2*d*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1)
)-9/2*a^2*b/c*x*d^2*e-3/2/c*a^2*b*x^2*d*e^2-9/2/c*b^3*e*arctan(c*x)^2*d^2*x
-3/2/c*b^3*e^2*arctan(c*x)^2*d*x^2+3/2/c^3*a*b^2*e^3*arctan(c*x)*x-9/c^2*b^
3*e*d^2*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+3*a*b^2*e^2*arctan(
c*x)^2*x^3*d+9/2*a*b^2*e*arctan(c*x)^2*x^2*d^2+3*a^2*b*e^2*arctan(c*x)*x^3*
d+9/2*a^2*b*e*arctan(c*x)*x^2*d^2-9/c^2*b^3*e*d^2*arctan(c*x)*ln(1-I*(1+I*c
```

```
*x)/(c^2*x^2+1)^(1/2))-3/c^3*b^3*e^2*d*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))*arct
an(c*x)^2+3/2/c^3*b^3*e^2*arctan(c*x)^2*ln(c^2*x^2+1)*d-1/2/c*a*b^2*e^3*arc
tan(c*x)*x^3-3/c*a*b^2*arctan(c*x)*ln(c^2*x^2+1)*d^3+9/2/c^2*a*b^2*e*d^2*ar
ctan(c*x)^2-3/c^3*a*b^2*e^2*arctan(c*x)*d+9/2/c^2*a*b^2*e*ln(c^2*x^2+1)*d^2
+3/2/c^3*a^2*b*ln(c^2*x^2+1)*d*e^2+9/2/c^2*a^2*b*e*arctan(c*x)*d^2+I/c^3*b^
3*e^2*d*arctan(c*x)^3-3/c^3*b^3*e^2*ln(2)*d*arctan(c*x)^2+3/4*I/c*a*b^2*d^3
*ln(c*x-I)^2+3/2*I/c*a*b^2*d^3*dilog(-1/2*I*(c*x+I))-3/2*I/c*a*b^2*d^3*dilo
g(1/2*I*(c*x-I))-3/4*I/c*a*b^2*d^3*ln(c*x+I)^2-3*I/c*b^3*d^3*arctan(c*x)*po
lylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-3*I/c^3*b^3*e^2*arctan(c*x)*d+9*I/c^2*b^3
*e*d^2*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+9/2*I/c^2*b^3*e*d^2*arctan(c*
x)^2+9*I/c^2*b^3*e*d^2*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/4*a^3*e^3*x^4 + a^3*d*e^2*x^3 + 7/32*b^3*d^3*arctan(c*x)^4/c + 112*b^3*c^
2*e^3*integrate(1/128*x^5*arctan(c*x)^3/(c^2*x^2 + 1), x) + 12*b^3*c^2*e^3*
integrate(1/128*x^5*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 384*
a*b^2*c^2*e^3*integrate(1/128*x^5*arctan(c*x)^2/(c^2*x^2 + 1), x) + 336*b^3
*c^2*d*e^2*integrate(1/128*x^4*arctan(c*x)^3/(c^2*x^2 + 1), x) + 12*b^3*c^2
*e^3*integrate(1/128*x^5*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 3
6*b^3*c^2*d*e^2*integrate(1/128*x^4*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^2
 + 1), x) + 1152*a*b^2*c^2*d*e^2*integrate(1/128*x^4*arctan(c*x)^2/(c^2*x^2
 + 1), x) + 336*b^3*c^2*d^2*e*integrate(1/128*x^3*arctan(c*x)^3/(c^2*x^2 +
1), x) + 48*b^3*c^2*d*e^2*integrate(1/128*x^4*arctan(c*x)*log(c^2*x^2 + 1)/
(c^2*x^2 + 1), x) + 36*b^3*c^2*d^2*e*integrate(1/128*x^3*arctan(c*x)*log(c^
2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 1152*a*b^2*c^2*d^2*e*integrate(1/128*x^3*a
rctan(c*x)^2/(c^2*x^2 + 1), x) + 112*b^3*c^2*d^3*integrate(1/128*x^2*arctan
(c*x)^3/(c^2*x^2 + 1), x) + 72*b^3*c^2*d^2*e*integrate(1/128*x^3*arctan(c*x
)*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 12*b^3*c^2*d^3*integrate(1/128*x^2*a
rctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 384*a*b^2*c^2*d^3*integra
te(1/128*x^2*arctan(c*x)^2/(c^2*x^2 + 1), x) + 48*b^3*c^2*d^3*integrate(1/1
28*x^2*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 3/2*a^3*d^2*e*x^2 +
 a*b^2*d^3*arctan(c*x)^3/c - 12*b^3*c*e^3*integrate(1/128*x^4*arctan(c*x)^2
/(c^2*x^2 + 1), x) + 3*b^3*c*e^3*integrate(1/128*x^4*log(c^2*x^2 + 1)^2/(c^
2*x^2 + 1), x) - 48*b^3*c*d*e^2*integrate(1/128*x^3*arctan(c*x)^2/(c^2*x^2
+ 1), x) + 12*b^3*c*d*e^2*integrate(1/128*x^3*log(c^2*x^2 + 1)^2/(c^2*x^2 +
```

```
 1), x) - 72*b^3*c*d^2*e*integrate(1/128*x^2*arctan(c*x)^2/(c^2*x^2 + 1), x
) + 18*b^3*c*d^2*e*integrate(1/128*x^2*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x)
 - 48*b^3*c*d^3*integrate(1/128*x*arctan(c*x)^2/(c^2*x^2 + 1), x) + 12*b^3*
c*d^3*integrate(1/128*x*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 9/2*(x^2*arc
tan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*a^2*b*d^2*e + 3/2*(2*x^3*arctan(c*x
) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*a^2*b*d*e^2 + 1/4*(3*x^4*arctan(c*x
) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a^2*b*e^3 + a^3*d^3*x + 11
2*b^3*e^3*integrate(1/128*x^3*arctan(c*x)^3/(c^2*x^2 + 1), x) + 12*b^3*e^3*
integrate(1/128*x^3*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 384*
a*b^2*e^3*integrate(1/128*x^3*arctan(c*x)^2/(c^2*x^2 + 1), x) + 336*b^3*d*e
^2*integrate(1/128*x^2*arctan(c*x)^3/(c^2*x^2 + 1), x) + 36*b^3*d*e^2*integ
rate(1/128*x^2*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 1152*a*b^
2*d*e^2*integrate(1/128*x^2*arctan(c*x)^2/(c^2*x^2 + 1), x) + 336*b^3*d^2*e
*integrate(1/128*x*arctan(c*x)^3/(c^2*x^2 + 1), x) + 36*b^3*d^2*e*integrate
(1/128*x*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 1152*a*b^2*d^2*
e*integrate(1/128*x*arctan(c*x)^2/(c^2*x^2 + 1), x) + 12*b^3*d^3*integrate(
1/128*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 3/2*(2*c*x*arctan(
c*x) - log(c^2*x^2 + 1))*a^2*b*d^3/c + 1/32*(b^3*e^3*x^4 + 4*b^3*d*e^2*x^3
 + 6*b^3*d^2*e*x^2 + 4*b^3*d^3*x)*arctan(c*x)^3 - 3/128*(b^3*e^3*x^4 + 4*b^3
*d*e^2*x^3 + 6*b^3*d^2*e*x^2 + 4*b^3*d^3*x)*arctan(c*x)*log(c^2*x^2 + 1)^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(a^3e^3x^3 + 3\,a^3de^2x^2 + 3\,a^3d^2ex + a^3d^3 + \left(b^3e^3x^3 + 3\,b^3de^2x^2 + 3\,b^3d^2ex + b^3d^3\right)\arctan\left(cx\right)^3 + 3\left(ab^2e^3x^3 + 3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(a^3*e^3*x^3 + 3*a^3*d*e^2*x^2 + 3*a^3*d^2*e*x + a^3*d^3 + (b^3*e^3
*x^3 + 3*b^3*d*e^2*x^2 + 3*b^3*d^2*e*x + b^3*d^3)*arctan(c*x)^3 + 3*(a*b^2*
e^3*x^3 + 3*a*b^2*d*e^2*x^2 + 3*a*b^2*d^2*e*x + a*b^2*d^3)*arctan(c*x)^2 +
3*(a^2*b*e^3*x^3 + 3*a^2*b*d*e^2*x^2 + 3*a^2*b*d^2*e*x + a^2*b*d^3)*arctan(
c*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\,\text{atan}\left(cx\right)\right)^3 \left(d + ex\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(a+b*atan(c*x))**3,x)
```

```
[Out] Integral((a + b*atan(c*x))**3*(d + e*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^3 (b \arctan(cx) + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(b*arctan(c*x) + a)^3, x)
```

3.16 $\int (d+ex)^2 \left(a+b\tan^{-1}(cx)\right)^3 dx$

Optimal. Leaf size=411

$$\frac{ib^2\left(3c^2d^2-e^2\right)\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^3}+\frac{b^3\left(3c^2d^2-e^2\right)\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^3}-\frac{3ib^3de\text{PolyLog}\left(2\right.}{c^2}$$

```
[Out] (a*b^2*e^2*x)/c^2 + (b^3*e^2*x*ArcTan[c*x])/c^2 - ((3*I)*b*d*e*(a + b*ArcTa
n[c*x])^2)/c^2 - (b*e^2*(a + b*ArcTan[c*x])^2)/(2*c^3) - (3*b*d*e*x*(a + b*
ArcTan[c*x])^2)/c - (b*e^2*x^2*(a + b*ArcTan[c*x])^2)/(2*c) + ((I/3)*(3*c^2
*d^2 - e^2)*(a + b*ArcTan[c*x])^3)/c^3 - (d*(d^2 - (3*e^2)/c^2)*(a + b*ArcT
an[c*x])^3)/(3*e) + ((d + e*x)^3*(a + b*ArcTan[c*x])^3)/(3*e) - (6*b^2*d*e*
(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c^2 + (b*(3*c^2*d^2 - e^2)*(a + b*A
rcTan[c*x])^2*Log[2/(1 + I*c*x)])/c^3 - (b^3*e^2*Log[1 + c^2*x^2])/(2*c^3)
 - ((3*I)*b^3*d*e*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^2 + (I*b^2*(3*c^2*d^2 - e
^2)*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^3 + (b^3*(3*c^2*d^
2 - e^2)*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*c^3)
```

Rubi [A] time = 0.774111, antiderivative size = 411, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {4864, 4846, 4920, 4854, 2402, 2315, 4852, 4916, 260, 4884, 4984, 4994, 6610}

$$\frac{ib^2\left(3c^2d^2-e^2\right)\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^3}+\frac{b^3\left(3c^2d^2-e^2\right)\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^3}-\frac{3ib^3de\text{PolyLog}\left(2\right.}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (a*b^2*e^2*x)/c^2 + (b^3*e^2*x*ArcTan[c*x])/c^2 - ((3*I)*b*d*e*(a + b*ArcTa
n[c*x])^2)/c^2 - (b*e^2*(a + b*ArcTan[c*x])^2)/(2*c^3) - (3*b*d*e*x*(a + b*
ArcTan[c*x])^2)/c - (b*e^2*x^2*(a + b*ArcTan[c*x])^2)/(2*c) + ((I/3)*(3*c^2
*d^2 - e^2)*(a + b*ArcTan[c*x])^3)/c^3 - (d*(d^2 - (3*e^2)/c^2)*(a + b*ArcT
an[c*x])^3)/(3*e) + ((d + e*x)^3*(a + b*ArcTan[c*x])^3)/(3*e) - (6*b^2*d*e*
(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c^2 + (b*(3*c^2*d^2 - e^2)*(a + b*A
rcTan[c*x])^2*Log[2/(1 + I*c*x)])/c^3 - (b^3*e^2*Log[1 + c^2*x^2])/(2*c^3)
 - ((3*I)*b^3*d*e*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^2 + (I*b^2*(3*c^2*d^2 - e
^2)*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^3 + (b^3*(3*c^2*d^
2 - e^2)*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*c^3)
```

Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
```

```
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4984

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int (d+ex)^2 \left(a+b\tan^{-1}(cx)\right)^3 dx = \frac{(d+ex)^3\left(a+b\tan^{-1}(cx)\right)^3}{3e} - \frac{(bc)\int \left(\frac{3de^2\left(a+b\tan^{-1}(cx)\right)^2}{c^2} + \frac{e^3x\left(a+b\tan^{-1}(cx)\right)^2}{c^2} + \frac{\left(c^2d^3-3d\right.}{}\right.}{e}$$

$$= \frac{(d+ex)^3\left(a+b\tan^{-1}(cx)\right)^3}{3e} - \frac{b\int \frac{\left(c^2d^3-3de^2+e\left(3c^2d^2-e^2\right)x\right)\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}dx}{ce} - \frac{(3bde)\int\left(\right.}{}$$

$$= -\frac{3bdex\left(a+b\tan^{-1}(cx)\right)^2}{c} - \frac{be^2x^2\left(a+b\tan^{-1}(cx)\right)^2}{2c} + \frac{(d+ex)^3\left(a+b\tan^{-1}(cx)\right)^3}{3e}$$

$$= -\frac{3ibde\left(a+b\tan^{-1}(cx)\right)^2}{c^2} - \frac{3bdex\left(a+b\tan^{-1}(cx)\right)^2}{c} - \frac{be^2x^2\left(a+b\tan^{-1}(cx)\right)^2}{2c} +$$

$$= \frac{ab^2e^2x}{c^2} - \frac{3ibde\left(a+b\tan^{-1}(cx)\right)^2}{c^2} - \frac{be^2\left(a+b\tan^{-1}(cx)\right)^2}{2c^3} - \frac{3bdex\left(a+b\tan^{-1}(cx\right.}{c}$$

$$= \frac{ab^2e^2x}{c^2} + \frac{b^3e^2x\tan^{-1}(cx)}{c^2} - \frac{3ibde\left(a+b\tan^{-1}(cx)\right)^2}{c^2} - \frac{be^2\left(a+b\tan^{-1}(cx)\right)^2}{2c^3} - \frac{3bc}{}$$

$$= \frac{ab^2e^2x}{c^2} + \frac{b^3e^2x\tan^{-1}(cx)}{c^2} - \frac{3ibde\left(a+b\tan^{-1}(cx)\right)^2}{c^2} - \frac{be^2\left(a+b\tan^{-1}(cx)\right)^2}{2c^3} - \frac{3bc}{}$$

$$= \frac{ab^2e^2x}{c^2} + \frac{b^3e^2x\tan^{-1}(cx)}{c^2} - \frac{3ibde\left(a+b\tan^{-1}(cx)\right)^2}{c^2} - \frac{be^2\left(a+b\tan^{-1}(cx)\right)^2}{2c^3} - \frac{3bc}{}$$

Mathematica [A] time = 1.11731, size = 621, normalized size = 1.51

$$\frac{18ab^2c^2d^2\left(\tan^{-1}(cx)\left((cx-i)\tan^{-1}(cx)+2\log\left(1+e^{2i\tan^{-1}(cx)}\right)\right)-i\text{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)\right)+6ab^2e^2\left(i\text{PolyLog}\left(2,\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^2*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (6*a^2*c^2*d*(a*c*d - 3*b*e)*x + 3*a^2*c^2*e*(2*a*c*d - b*e)*x^2 + 2*a^3*c^
3*e^2*x^3 + 18*a^2*b*c*d*e*ArcTan[c*x] + 6*a^2*b*c^3*x*(3*d^2 + 3*d*e*x + e
^2*x^2)*ArcTan[c*x] - 3*a^2*b*(3*c^2*d^2 - e^2)*Log[1 + c^2*x^2] + 18*a*b^2
*c*d*e*(-2*c*x*ArcTan[c*x] + (1 + c^2*x^2)*ArcTan[c*x]^2 + Log[1 + c^2*x^2]
) + 18*a*b^2*c^2*d^2*(ArcTan[c*x]*((-I + c*x)*ArcTan[c*x] + 2*Log[1 + E^((2
*I)*ArcTan[c*x])]) - I*PolyLog[2, -E^((2*I)*ArcTan[c*x])]) + 6*a*b^2*e^2*(c
```

```
*x + (I + c^3*x^3)*ArcTan[c*x]^2 - ArcTan[c*x]*(1 + c^2*x^2 + 2*Log[1 + E^(
(2*I)*ArcTan[c*x])]) + I*PolyLog[2, -E^((2*I)*ArcTan[c*x])]) + 6*b^3*c*d*e*
(ArcTan[c*x]*((3*I - 3*c*x)*ArcTan[c*x] + (1 + c^2*x^2)*ArcTan[c*x]^2 - 6*L
og[1 + E^((2*I)*ArcTan[c*x])]) + (3*I)*PolyLog[2, -E^((2*I)*ArcTan[c*x])])
+ b^3*e^2*(6*c*x*ArcTan[c*x] - 3*ArcTan[c*x]^2 - 3*c^2*x^2*ArcTan[c*x]^2 +
(2*I)*ArcTan[c*x]^3 + 2*c^3*x^3*ArcTan[c*x]^3 - 6*ArcTan[c*x]^2*Log[1 + E^(
(2*I)*ArcTan[c*x])] - 3*Log[1 + c^2*x^2] + (6*I)*ArcTan[c*x]*PolyLog[2, -E^
((2*I)*ArcTan[c*x])] - 3*PolyLog[3, -E^((2*I)*ArcTan[c*x])]) + 3*b^3*c^2*d^
2*(2*ArcTan[c*x]^2*((-I + c*x)*ArcTan[c*x] + 3*Log[1 + E^((2*I)*ArcTan[c*x]
)]) - (6*I)*ArcTan[c*x]*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + 3*PolyLog[3, -
E^((2*I)*ArcTan[c*x])]))/(6*c^3)
```

Maple [C] time = 2.606, size = 3022, normalized size = 7.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(a+b*arctan(c*x))^3,x)
```

```
[Out] 1/3*a^3/e*d^3-1/2/c^3*b^3*e^2*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+1/c^3*b^3
*e^2*ln((1+I*c*x)^2/(c^2*x^2+1)+1)-1/2/c^3*b^3*e^2*arctan(c*x)^2+3/2/c*b^3*
polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))*d^2+b^3*arctan(c*x)^3*x*d^2+1/3*b^3*e^2
*arctan(c*x)^3*x^3+a*b^2*e^2*x/c^2+b^3*e^2*x*arctan(c*x)/c^2-6/c^2*b^3*e*d*
arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-6/c^2*b^3*e*d*arctan(c*x)*l
n(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I/c^3*b^3*e^2*arctan(c*x)*polylog(2,-(1+
I*c*x)^2/(c^2*x^2+1))+1/c^3*a*b^2*e^2*arctan(c*x)*ln(c^2*x^2+1)+3*a^2*b*e*a
rctan(c*x)*x^2*d+3*a*b^2*e*arctan(c*x)^2*x^2*d-3/c*a*b^2*arctan(c*x)*ln(c^2
*x^2+1)*d^2+3/c^2*a*b^2*e*ln(c^2*x^2+1)*d-3/c*b^3*e*arctan(c*x)^2*d*x-1/c*a
*b^2*e^2*arctan(c*x)*x^2+6*I/c^2*b^3*e*d*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1
/2))+3/4*I/c*a*b^2*ln(c*x-I)^2*d^2+3/2*I/c*a*b^2*dilog(-1/2*I*(c*x+I))*d^2-
3/4*I/c*a*b^2*ln(c*x+I)^2*d^2-3/2*I/c*a*b^2*dilog(1/2*I*(c*x-I))*d^2-1/4*I/
c^3*a*b^2*ln(c*x-I)^2*e^2-1/2*I/c^3*a*b^2*dilog(-1/2*I*(c*x+I))*e^2+1/4*I/c
^3*a*b^2*ln(c*x+I)^2*e^2+1/2*I/c^3*a*b^2*dilog(1/2*I*(c*x-I))*e^2-3*I/c*b^3
*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))*d^2+3*I/c^2*b^3*e*d*arctan
(c*x)^2+6*I/c^2*b^3*e*d*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-3*a^2*b/c*x*
d*e+a^3*e*x^2*d+3/c^2*a^2*b*e*arctan(c*x)*d+3/c^2*a*b^2*e*d*arctan(c*x)^2+3
*a^2*b*arctan(c*x)*x*d^2+3*a*b^2*arctan(c*x)^2*x*d^2+3/c*b^3*ln((1+I*c*x)/(
c^2*x^2+1)^(1/2))*arctan(c*x)^2*d^2-3/2/c*b^3*arctan(c*x)^2*ln(c^2*x^2+1)*d
^2-1/2/c*b^3*e^2*arctan(c*x)^2*x^2+1/2/c^3*a^2*b*ln(c^2*x^2+1)*e^2-1/c^3*b^
3*e^2*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))*arctan(c*x)^2+1/2/c^3*b^3*e^2*arctan(
c*x)^2*ln(c^2*x^2+1)-1/c^3*a*b^2*e^2*arctan(c*x)+1/c^2*b^3*e*arctan(c*x)^3*
```

```
d-1/c^3*b^3*e^2*ln(2)*arctan(c*x)^2+3/c*b^3*d^2*ln(2)*arctan(c*x)^2-3/2/c*a
^2*b*ln(c^2*x^2+1)*d^2-I/c^3*b^3*e^2*arctan(c*x)+1/3*I/c^3*b^3*e^2*arctan(c
*x)^3-I/c*b^3*arctan(c*x)^3*d^2-1/2/c*a^2*b*x^2*e^2+b^3*e*arctan(c*x)^3*x^2
*d+a^2*b*e^2*arctan(c*x)*x^3+a*b^2*e^2*arctan(c*x)^2*x^3+a^3*x*d^2+1/3*a^3*
e^2*x^3+1/2*I/c^3*b^3*e^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1
+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*arctan(c*x)^2+1/4*I/c^3*b^3*e^2*Pi*csgn(I*(1+
I*c*x)/(c^2*x^2+1)^(1/2))^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*arctan(c*x)^2-3
/4*I/c*b^3*d^2*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2*csgn(I*(1+I*c*x)^2/
(c^2*x^2+1))*arctan(c*x)^2+3/4*I/c*b^3*d^2*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1
))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*arctan(c
*x)^2+3/4*I/c*b^3*d^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I*((1+I
*c*x)^2/(c^2*x^2+1)+1)^2)*arctan(c*x)^2+3/4*I/c*b^3*d^2*Pi*csgn(I/((1+I*c*x
)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+
1)+1)^2)^2*arctan(c*x)^2+3/2*I/c*b^3*d^2*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1
/2))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2*arctan(c*x)^2-3/2*I/c*b^3*d^2*Pi*csg
n(I*((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*ar
ctan(c*x)^2-1/4*I/c^3*b^3*e^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn
(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*arctan(c*x)^2-1/2*I/c^3*b^3*e^2*Pi*csgn(I
*(1+I*c*x)/(c^2*x^2+1)^(1/2))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2*arctan(c*x)
^2-1/4*I/c^3*b^3*e^2*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c
*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*arctan(c*x)^2-1/4*I/c^3*
b^3*e^2*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(
(1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2*arctan(c*x)^2+1/2*I/c^3*a*b^2*ln(c*x+I)*ln(
1/2*I*(c*x-I))*e^2-1/4*I/c^3*b^3*e^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^
2)^3*arctan(c*x)^2+1/4*I/c^3*b^3*e^2*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3*a
rctan(c*x)^2-6/c*a*b^2*e*arctan(c*x)*d*x+1/4*I/c^3*b^3*e^2*Pi*csgn(I*(1+I*c
*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*arctan(c*x)^2-3/4*I/c*b^
3*d^2*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*ar
ctan(c*x)^2+3/4*I/c*b^3*d^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3*arct
an(c*x)^2-3/4*I/c*b^3*d^2*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3*arctan(c*x)^
2-3/2*I/c*a*b^2*ln(c^2*x^2+1)*ln(c*x-I)*d^2+3/2*I/c*a*b^2*ln(c*x-I)*ln(-1/2
*I*(c*x+I))*d^2+3/2*I/c*a*b^2*ln(c^2*x^2+1)*ln(c*x+I)*d^2-3/2*I/c*a*b^2*ln(
c*x+I)*ln(1/2*I*(c*x-I))*d^2+1/2*I/c^3*a*b^2*ln(c^2*x^2+1)*ln(c*x-I)*e^2-1/
2*I/c^3*a*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))*e^2-1/2*I/c^3*a*b^2*ln(c^2*x^2+1
)*ln(c*x+I)*e^2-3/4*I/c*b^3*d^2*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*cs
gn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(
c^2*x^2+1)+1)^2)*arctan(c*x)^2+1/4*I/c^3*b^3*e^2*Pi*csgn(I/((1+I*c*x)^2/(c^
2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+
1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*arctan(c*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/3*a^3*e^2*x^3 + 7/32*b^3*d^2*arctan(c*x)^4/c + 28*b^3*c^2*e^2*integrate(1
/32*x^4*arctan(c*x)^3/(c^2*x^2 + 1), x) + 3*b^3*c^2*e^2*integrate(1/32*x^4*
arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 96*a*b^2*c^2*e^2*integra
te(1/32*x^4*arctan(c*x)^2/(c^2*x^2 + 1), x) + 56*b^3*c^2*d*e*integrate(1/32
*x^3*arctan(c*x)^3/(c^2*x^2 + 1), x) + 4*b^3*c^2*e^2*integrate(1/32*x^4*arc
tan(c*x)*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 6*b^3*c^2*d*e*integrate(1/32*
x^3*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 192*a*b^2*c^2*d*e*in
tegrate(1/32*x^3*arctan(c*x)^2/(c^2*x^2 + 1), x) + 28*b^3*c^2*d^2*integrate
(1/32*x^2*arctan(c*x)^3/(c^2*x^2 + 1), x) + 12*b^3*c^2*d*e*integrate(1/32*x
^3*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 3*b^3*c^2*d^2*integrate
(1/32*x^2*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 96*a*b^2*c^2*d
^2*integrate(1/32*x^2*arctan(c*x)^2/(c^2*x^2 + 1), x) + 12*b^3*c^2*d^2*inte
grate(1/32*x^2*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + a^3*d*e*x^2
 + a*b^2*d^2*arctan(c*x)^3/c - 4*b^3*c*e^2*integrate(1/32*x^3*arctan(c*x)^2
/(c^2*x^2 + 1), x) + b^3*c*e^2*integrate(1/32*x^3*log(c^2*x^2 + 1)^2/(c^2*x
^2 + 1), x) - 12*b^3*c*d*e*integrate(1/32*x^2*arctan(c*x)^2/(c^2*x^2 + 1),
x) + 3*b^3*c*d*e*integrate(1/32*x^2*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) -
12*b^3*c*d^2*integrate(1/32*x*arctan(c*x)^2/(c^2*x^2 + 1), x) + 3*b^3*c*d^2
*integrate(1/32*x*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 3*(x^2*arctan(c*x)
 - c*(x/c^2 - arctan(c*x)/c^3))*a^2*b*d*e + 1/2*(2*x^3*arctan(c*x) - c*(x^2
/c^2 - log(c^2*x^2 + 1)/c^4))*a^2*b*e^2 + a^3*d^2*x + 28*b^3*e^2*integrate(
1/32*x^2*arctan(c*x)^3/(c^2*x^2 + 1), x) + 3*b^3*e^2*integrate(1/32*x^2*arc
tan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 96*a*b^2*e^2*integrate(1/32
*x^2*arctan(c*x)^2/(c^2*x^2 + 1), x) + 56*b^3*d*e*integrate(1/32*x*arctan(c
*x)^3/(c^2*x^2 + 1), x) + 6*b^3*d*e*integrate(1/32*x*arctan(c*x)*log(c^2*x^
2 + 1)^2/(c^2*x^2 + 1), x) + 192*a*b^2*d*e*integrate(1/32*x*arctan(c*x)^2/(
c^2*x^2 + 1), x) + 3*b^3*d^2*integrate(1/32*arctan(c*x)*log(c^2*x^2 + 1)^2/
(c^2*x^2 + 1), x) + 3/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a^2*b*d^2/c
+ 1/24*(b^3*e^2*x^3 + 3*b^3*d*e*x^2 + 3*b^3*d^2*x)*arctan(c*x)^3 - 1/32*(b^
3*e^2*x^3 + 3*b^3*d*e*x^2 + 3*b^3*d^2*x)*arctan(c*x)*log(c^2*x^2 + 1)^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(a^3e^2x^2 + 2\,a^3dex + a^3d^2 + \left(b^3e^2x^2 + 2\,b^3dex + b^3d^2\right)\arctan(cx)^3 + 3\left(ab^2e^2x^2 + 2\,ab^2dex + ab^2d^2\right)\arctan(c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(a^3*e^2*x^2 + 2*a^3*d*e*x + a^3*d^2 + (b^3*e^2*x^2 + 2*b^3*d*e*x +
 b^3*d^2)*arctan(c*x)^3 + 3*(a*b^2*e^2*x^2 + 2*a*b^2*d*e*x + a*b^2*d^2)*arc
tan(c*x)^2 + 3*(a^2*b*e^2*x^2 + 2*a^2*b*d*e*x + a^2*b*d^2)*arctan(c*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{atan}(cx))^3 (d + ex)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(a+b*atan(c*x))**3,x)
```

```
[Out] Integral((a + b*atan(c*x))**3*(d + e*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^2 (b \arctan(cx) + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(b*arctan(c*x) + a)^3, x)
```

3.17 $\int (d+ex)\left(a+b\tan^{-1}(cx)\right)^3\,dx$

Optimal. Leaf size=264

$$\frac{3ib^2d\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c}-\frac{3ib^3e\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^2}+\frac{3b^3d\,\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c}-\frac{3b^2e\log}{}$$

```
[Out] (((-3*I)/2)*b*e*(a + b*ArcTan[c*x])^2)/c^2 - (3*b*e*x*(a + b*ArcTan[c*x])^2
)/(2*c) + (I*d*(a + b*ArcTan[c*x])^3)/c - ((d^2 - e^2/c^2)*(a + b*ArcTan[c*
x])^3)/(2*e) + ((d + e*x)^2*(a + b*ArcTan[c*x])^3)/(2*e) - (3*b^2*e*(a + b*
ArcTan[c*x])*Log[2/(1 + I*c*x)])/c^2 + (3*b*d*(a + b*ArcTan[c*x])^2*Log[2/(
1 + I*c*x)])/c - (((3*I)/2)*b^3*e*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^2 + ((3*
I)*b^2*d*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/c + (3*b^3*d*Po
lyLog[3, 1 - 2/(1 + I*c*x)])/(2*c)
```

Rubi [A] time = 0.580084, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {4864, 4846, 4920, 4854, 2402, 2315, 4984, 4884, 4994, 6610}

$$\frac{3ib^2d\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c}-\frac{3ib^3e\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^2}+\frac{3b^3d\,\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c}-\frac{3b^2e\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (((-3*I)/2)*b*e*(a + b*ArcTan[c*x])^2)/c^2 - (3*b*e*x*(a + b*ArcTan[c*x])^2
)/(2*c) + (I*d*(a + b*ArcTan[c*x])^3)/c - ((d^2 - e^2/c^2)*(a + b*ArcTan[c*
x])^3)/(2*e) + ((d + e*x)^2*(a + b*ArcTan[c*x])^3)/(2*e) - (3*b^2*e*(a + b*
ArcTan[c*x])*Log[2/(1 + I*c*x)])/c^2 + (3*b*d*(a + b*ArcTan[c*x])^2*Log[2/(
1 + I*c*x)])/c - (((3*I)/2)*b^3*e*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^2 + ((3*
I)*b^2*d*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/c + (3*b^3*d*Po
lyLog[3, 1 - 2/(1 + I*c*x)])/(2*c)
```

Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
```

```
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4984

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
```

```
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int (d+ex)\left(a+b\tan^{-1}(cx)\right)^3\,dx = \frac{(d+ex)^2\left(a+b\tan^{-1}(cx)\right)^3}{2e} - \frac{(3bc)\int\left(\frac{e^2\left(a+b\tan^{-1}(cx)\right)^2}{c^2} + \frac{\left(c^2d^2-e^2+2c^2dex\right)\left(a+b\tan^{-1}(cx)\right)}{c^2\left(1+c^2x^2\right)}\right.}{2e}$$

$$= \frac{(d+ex)^2\left(a+b\tan^{-1}(cx)\right)^3}{2e} - \frac{(3b)\int\frac{\left(c^2d^2-e^2+2c^2dex\right)\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}\,dx}{2ce} - \frac{(3be)\int\left(a+\right.}{}$$

$$= -\frac{3bex\left(a+b\tan^{-1}(cx)\right)^2}{2c} + \frac{(d+ex)^2\left(a+b\tan^{-1}(cx)\right)^3}{2e} - \frac{(3b)\int\left(\frac{c^2d^2\left(1-\frac{e^2}{c^2d^2}\right)\left(a+b\tan\right.}{1+c^2x^2}\right.}{}$$

$$= -\frac{3ibe\left(a+b\tan^{-1}(cx)\right)^2}{2c^2} - \frac{3bex\left(a+b\tan^{-1}(cx)\right)^2}{2c} + \frac{(d+ex)^2\left(a+b\tan^{-1}(cx)\right)^3}{2e}$$

$$= -\frac{3ibe\left(a+b\tan^{-1}(cx)\right)^2}{2c^2} - \frac{3bex\left(a+b\tan^{-1}(cx)\right)^2}{2c} + \frac{id\left(a+b\tan^{-1}(cx)\right)^3}{c} - \frac{\left(d^2-\right.}{}$$

$$= -\frac{3ibe\left(a+b\tan^{-1}(cx)\right)^2}{2c^2} - \frac{3bex\left(a+b\tan^{-1}(cx)\right)^2}{2c} + \frac{id\left(a+b\tan^{-1}(cx)\right)^3}{c} - \frac{\left(d^2-\right.}{}$$

$$= -\frac{3ibe\left(a+b\tan^{-1}(cx)\right)^2}{2c^2} - \frac{3bex\left(a+b\tan^{-1}(cx)\right)^2}{2c} + \frac{id\left(a+b\tan^{-1}(cx)\right)^3}{c} - \frac{\left(d^2-\right.}{}$$

$$= -\frac{3ibe\left(a+b\tan^{-1}(cx)\right)^2}{2c^2} - \frac{3bex\left(a+b\tan^{-1}(cx)\right)^2}{2c} + \frac{id\left(a+b\tan^{-1}(cx)\right)^3}{c} - \frac{\left(d^2-\right.}{}$$

Mathematica [A] time = 0.594318, size = 342, normalized size = 1.3

$$\frac{6ab^2cd\left(\tan^{-1}(cx)\left((cx-i)\tan^{-1}(cx)+2\log\left(1+e^{2i\tan^{-1}(cx)}\right)\right)-i\text{PolyLog}\left(2,-e^{2i\tan^{-1}(cx)}\right)\right)+b^3e\left(3i\text{PolyLog}\left(2,-e^{2i\tan^{-1}}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (a^2*c*(2*a*c*d - 3*b*e)*x + a^3*c^2*e*x^2 + 3*a^2*b*e*ArcTan[c*x] + 3*a^2*
b*c^2*x*(2*d + e*x)*ArcTan[c*x] - 3*a^2*b*c*d*Log[1 + c^2*x^2] + 3*a*b^2*e*
(-2*c*x*ArcTan[c*x] + (1 + c^2*x^2)*ArcTan[c*x]^2 + Log[1 + c^2*x^2]) + 6*a
*b^2*c*d*(ArcTan[c*x]*((-I + c*x)*ArcTan[c*x] + 2*Log[1 + E^((2*I)*ArcTan[c
*x])]) - I*PolyLog[2, -E^((2*I)*ArcTan[c*x])]) + b^3*e*(ArcTan[c*x]*((3*I -
 3*c*x)*ArcTan[c*x] + (1 + c^2*x^2)*ArcTan[c*x]^2 - 6*Log[1 + E^((2*I)*ArcT
an[c*x])]) + (3*I)*PolyLog[2, -E^((2*I)*ArcTan[c*x])]) + b^3*c*d*(2*ArcTan[
c*x]^2*((-I + c*x)*ArcTan[c*x] + 3*Log[1 + E^((2*I)*ArcTan[c*x])]) - (6*I)*
ArcTan[c*x]*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + 3*PolyLog[3, -E^((2*I)*Arc
Tan[c*x])]))/(2*c^2)
```

Maple [C] time = 0.813, size = 7462, normalized size = 28.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(a+b*arctan(c*x))^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

```
[Out] 7/32*b^3*d*arctan(c*x)^4/c + 56*b^3*c^2*e*integrate(1/64*x^3*arctan(c*x)^3/
(c^2*x^2 + 1), x) + 6*b^3*c^2*e*integrate(1/64*x^3*arctan(c*x)*log(c^2*x^2
+ 1)^2/(c^2*x^2 + 1), x) + 192*a*b^2*c^2*e*integrate(1/64*x^3*arctan(c*x)^2
/(c^2*x^2 + 1), x) + 56*b^3*c^2*d*integrate(1/64*x^2*arctan(c*x)^3/(c^2*x^2
 + 1), x) + 12*b^3*c^2*e*integrate(1/64*x^3*arctan(c*x)*log(c^2*x^2 + 1)/(c
^2*x^2 + 1), x) + 6*b^3*c^2*d*integrate(1/64*x^2*arctan(c*x)*log(c^2*x^2 +
1)^2/(c^2*x^2 + 1), x) + 192*a*b^2*c^2*d*integrate(1/64*x^2*arctan(c*x)^2/(
c^2*x^2 + 1), x) + 24*b^3*c^2*d*integrate(1/64*x^2*arctan(c*x)*log(c^2*x^2
+ 1)/(c^2*x^2 + 1), x) + 1/2*a^3*e*x^2 + a*b^2*d*arctan(c*x)^3/c - 12*b^3*c
*e*integrate(1/64*x^2*arctan(c*x)^2/(c^2*x^2 + 1), x) + 3*b^3*c*e*integrate
(1/64*x^2*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) - 24*b^3*c*d*integrate(1/64*
x*arctan(c*x)^2/(c^2*x^2 + 1), x) + 6*b^3*c*d*integrate(1/64*x*log(c^2*x^2
+ 1)^2/(c^2*x^2 + 1), x) + 3/2*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^
3))*a^2*b*e + a^3*d*x + 56*b^3*e*integrate(1/64*x*arctan(c*x)^3/(c^2*x^2 +
1), x) + 6*b^3*e*integrate(1/64*x*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^2 +
 1), x) + 192*a*b^2*e*integrate(1/64*x*arctan(c*x)^2/(c^2*x^2 + 1), x) + 6*
b^3*d*integrate(1/64*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 3/2
*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a^2*b*d/c + 1/16*(b^3*e*x^2 + 2*b^3
*d*x)*arctan(c*x)^3 - 3/64*(b^3*e*x^2 + 2*b^3*d*x)*arctan(c*x)*log(c^2*x^2
+ 1)^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(a^3ex + a^3d + \left(b^3ex + b^3d\right)\arctan(cx)^3 + 3\left(ab^2ex + ab^2d\right)\arctan(cx)^2 + 3\left(a^2bex + a^2bd\right)\arctan(cx), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(a^3*e*x + a^3*d + (b^3*e*x + b^3*d)*arctan(c*x)^3 + 3*(a*b^2*e*x +
 a*b^2*d)*arctan(c*x)^2 + 3*(a^2*b*e*x + a^2*b*d)*arctan(c*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\operatorname{atan}(cx))^3 (d + ex)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*atan(c*x))**3,x)
```

```
[Out] Integral((a + b*atan(c*x))**3*(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex + d)(b \arctan(cx) + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(b*arctan(c*x) + a)^3, x)
```

3.18 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{d+ex}\,dx$

Optimal. Leaf size=320

$$\frac{3b^2\left(a+b\tan^{-1}(cx)\right)\text{PolyLog}\left(3,1-\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{2e}-\frac{3b^2\text{PolyLog}\left(3,1-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)}{2e}-\frac{3ib\left(a+b\tan^{-1}(cx\right.}{}$$

```
[Out] -(((a + b*ArcTan[c*x])^3*Log[2/(1 - I*c*x)])/e) + ((a + b*ArcTan[c*x])^3*Lo
g[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e + (((3*I)/2)*b*(a + b*ArcTa
n[c*x])^2*PolyLog[2, 1 - 2/(1 - I*c*x)])/e - (((3*I)/2)*b*(a + b*ArcTan[c*x
])^2*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e - (3*b^2*
(a + b*ArcTan[c*x])*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e) + (3*b^2*(a + b*Ar
cTan[c*x])*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e)
 - (((3*I)/4)*b^3*PolyLog[4, 1 - 2/(1 - I*c*x)])/e + (((3*I)/4)*b^3*PolyLog
[4, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e
```

Rubi [A] time = 0.0568978, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {4860}

$$\frac{3b^2\left(a+b\tan^{-1}(cx)\right)\text{PolyLog}\left(3,1-\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{2e}-\frac{3b^2\text{PolyLog}\left(3,1-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)}{2e}-\frac{3ib\left(a+b\tan^{-1}(cx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^3/(d + e*x),x]
```

```
[Out] -(((a + b*ArcTan[c*x])^3*Log[2/(1 - I*c*x)])/e) + ((a + b*ArcTan[c*x])^3*Lo
g[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e + (((3*I)/2)*b*(a + b*ArcTa
n[c*x])^2*PolyLog[2, 1 - 2/(1 - I*c*x)])/e - (((3*I)/2)*b*(a + b*ArcTan[c*x
])^2*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e - (3*b^2*
(a + b*ArcTan[c*x])*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e) + (3*b^2*(a + b*Ar
cTan[c*x])*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e)
 - (((3*I)/4)*b^3*PolyLog[4, 1 - 2/(1 - I*c*x)])/e + (((3*I)/4)*b^3*PolyLog
[4, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e
```

Rule 4860

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^3/((d_) + (e_.)*(x_)), x_Symbol] :>
-Simp[((a + b*ArcTan[c*x])^3*Log[2/(1 - I*c*x)])/e, x] + (Simp[((a + b*ArcT
```

```
an[c*x])^3*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x] + Simp[(3*
I*b*(a + b*ArcTan[c*x])^2*PolyLog[2, 1 - 2/(1 - I*c*x)])/(2*e), x] - Simp[(
3*I*b*(a + b*ArcTan[c*x])^2*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1
 - I*c*x))])/(2*e), x] - Simp[(3*b^2*(a + b*ArcTan[c*x])*PolyLog[3, 1 - 2/(1
 - I*c*x)])/(2*e), x] + Simp[(3*b^2*(a + b*ArcTan[c*x])*PolyLog[3, 1 - (2*c
*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e), x] - Simp[(3*I*b^3*PolyLog[4
, 1 - 2/(1 - I*c*x)])/(4*e), x] + Simp[(3*I*b^3*PolyLog[4, 1 - (2*c*(d + e*
x))/((c*d + I*e)*(1 - I*c*x))])/(4*e), x]) /; FreeQ[{a, b, c, d, e}, x] &&
NeQ[c^2*d^2 + e^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{d+ex}\,dx = -\frac{\left(a+b\tan^{-1}(cx)\right)^3\log\left(\frac{2}{1-icx}\right)}{e} + \frac{\left(a+b\tan^{-1}(cx)\right)^3\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e} + \frac{3ib\left(a+b\tan^{-1}(cx)\right.}{2e}$$

Mathematica [F] time = 180.004, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x])^3/(d + e*x),x]
```

```
[Out] $Aborted
```

Maple [C] time = 0.573, size = 2616, normalized size = 8.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^3/(e*x+d),x)
```

```
[Out] b^3*ln(c*e*x+c*d)/e*arctan(c*x)^3-b^3/e*arctan(c*x)^3*ln(-I*(1+I*c*x)^2/(c^
2*x^2+1)*e+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+d*c)-3/2*b^3/e*arctan(c*x)*polyl
og(3,-(1+I*c*x)^2/(c^2*x^2+1))+b^3*arctan(c*x)^3*ln(1-(I*e-d*c)/(d*c+I*e)*(
1+I*c*x)^2/(c^2*x^2+1))/(e+I*d*c)+3/2*b^3*arctan(c*x)*polylog(3,(I*e-d*c)/(
d*c+I*e)*(1+I*c*x)^2/(c^2*x^2+1))/(e+I*d*c)+3/2*a*b^2*polylog(3,(I*e-d*c)/(
```

```
d*c+I*e)*(1+I*c*x)^2/(c^2*x^2+1))/(e+I*d*c)-3/2*a*b^2/e*polylog(3,-(1+I*c*x
)^2/(c^2*x^2+1))-3/4*I*b^3/e*polylog(4,-(1+I*c*x)^2/(c^2*x^2+1))+3/4*I*b^3*
polylog(4,(I*e-d*c)/(d*c+I*e)*(1+I*c*x)^2/(c^2*x^2+1))/(e+I*d*c)+a^3*ln(c*e
*x+c*d)/e-3*I*c*a*b^2/e*d/(d*c-I*e)*arctan(c*x)*polylog(2,(I*e-d*c)/(d*c+I*
e)*(1+I*c*x)^2/(c^2*x^2+1))+3/2*I*a*b^2/e*arctan(c*x)^2*Pi*csgn(I/((1+I*c*x
)^2/(c^2*x^2+1)+1))*csgn(I*(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)^2/(c
^2*x^2+1)+I*e+d*c)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*(-I*(1+I*c*x)^2/(c^2
*x^2+1)*e+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+d*c))+3*a*b^2*ln(c*e*x+c*d)/e*arc
tan(c*x)^2-3*a*b^2/e*arctan(c*x)^2*ln(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I
*c*x)^2/(c^2*x^2+1)+I*e+d*c)+3*a*b^2*arctan(c*x)^2*ln(1-(I*e-d*c)/(d*c+I*e)
*(1+I*c*x)^2/(c^2*x^2+1))/(e+I*d*c)+3*a^2*b*ln(c*e*x+c*d)/e*arctan(c*x)+3/2
*I*b^3/e*arctan(c*x)^2*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-3/2*I*b^3*arctan
(c*x)^2*polylog(2,(I*e-d*c)/(d*c+I*e)*(1+I*c*x)^2/(c^2*x^2+1))/(e+I*d*c)+3/
2*I*a^2*b/e*dilog((I*e-e*c*x)/(d*c+I*e))-3/2*I*a^2*b/e*dilog((I*e+e*c*x)/(I
*e-d*c))+3/2*c*a*b^2/e*d/(d*c-I*e)*polylog(3,(I*e-d*c)/(d*c+I*e)*(1+I*c*x)^
2/(c^2*x^2+1))+c*b^3/e*d/(d*c-I*e)*arctan(c*x)^3*ln(1-(I*e-d*c)/(d*c+I*e)*(
1+I*c*x)^2/(c^2*x^2+1))+3/2*c*b^3/e*d/(d*c-I*e)*arctan(c*x)*polylog(3,(I*e-
d*c)/(d*c+I*e)*(1+I*c*x)^2/(c^2*x^2+1))+3/2*I*a*b^2/e*arctan(c*x)^2*Pi*csgn
(I*(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+d*c)/((1+I
*c*x)^2/(c^2*x^2+1)+1))^3-1/2*I*b^3/e*arctan(c*x)^3*Pi*csgn(I*(-I*(1+I*c*x)
^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+d*c)/((1+I*c*x)^2/(c^2*x^2
+1)+1))^2*csgn(I*(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)^2/(c^2*x^2+1)+
I*e+d*c))-3/2*I*a^2*b*ln(c*e*x+c*d)/e*ln((I*e+e*c*x)/(I*e-d*c))+3*I*a*b^2/e
*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-3*I*a*b^2*arctan(c*x)*poly
log(2,(I*e-d*c)/(d*c+I*e)*(1+I*c*x)^2/(c^2*x^2+1))/(e+I*d*c)+3*c*a*b^2/e*d/
(d*c-I*e)*arctan(c*x)^2*ln(1-(I*e-d*c)/(d*c+I*e)*(1+I*c*x)^2/(c^2*x^2+1))-3
/2*I*a*b^2/e*arctan(c*x)^2*Pi*csgn(I*(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I
*c*x)^2/(c^2*x^2+1)+I*e+d*c)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*csgn(I*(-I*(1+I
*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+d*c))+1/2*I*b^3/e*arc
tan(c*x)^3*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*(-I*(1+I*c*x)^2/(c
^2*x^2+1)*e+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+d*c)/((1+I*c*x)^2/(c^2*x^2+1)+1
))*csgn(I*(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+d*c
))-3/2*I*a*b^2/e*arctan(c*x)^2*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(
I*(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+d*c)/((1+I*
c*x)^2/(c^2*x^2+1)+1))^2-3/2*I*c*b^3/e*d/(d*c-I*e)*arctan(c*x)^2*polylog(2,
(I*e-d*c)/(d*c+I*e)*(1+I*c*x)^2/(c^2*x^2+1))+1/2*I*b^3/e*arctan(c*x)^3*Pi*c
sgn(I*(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+d*c)/((
1+I*c*x)^2/(c^2*x^2+1)+1))^3+3/2*I*a^2*b*ln(c*e*x+c*d)/e*ln((I*e-e*c*x)/(d*
c+I*e))-1/2*I*b^3/e*arctan(c*x)^3*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*cs
gn(I*(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+d*c)/((1
+I*c*x)^2/(c^2*x^2+1)+1))^2+3/4*I*c*b^3/e*d/(d*c-I*e)*polylog(4,(I*e-d*c)/(
d*c+I*e)*(1+I*c*x)^2/(c^2*x^2+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^3 \log(ex+d)}{e} + \int \frac{28\,b^3 \arctan(cx)^3 + 3\,b^3 \arctan(cx) \log\left(c^2x^2+1\right)^2 + 96\,ab^2 \arctan(cx)^2 + 96\,a^2 b \arctan(cx)}{32\,(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] a^3*log(e*x + d)/e + integrate(1/32*(28*b^3*arctan(c*x)^3 + 3*b^3*arctan(c*
x)*log(c^2*x^2 + 1)^2 + 96*a*b^2*arctan(c*x)^2 + 96*a^2*b*arctan(c*x))/(e*x
 + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3 \arctan(cx)^3 + 3\,ab^2 \arctan(cx)^2 + 3\,a^2 b \arctan(cx) + a^3}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x) +
 a^3)/(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{atan}(cx))^3}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**3/(e*x+d),x)
```

```
[Out] Integral((a + b*atan(c*x))**3/(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(cx) + a)^3}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^3/(e*x + d), x)
```

3.19 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{(d+ex)^2}\,dx$

Optimal. Leaf size=499

$$\frac{3ib^2c\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^2d^2+e^2}+\frac{3ib^2c\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^2d^2+e^2}-\frac{3ib^2c\left(a+b\tan^{-1}(cx)\right)}{c^2}$$

```
[Out] (I*c*(a + b*ArcTan[c*x])^3)/(c^2*d^2 + e^2) + (c^2*d*(a + b*ArcTan[c*x])^3)
/(e*(c^2*d^2 + e^2)) - (a + b*ArcTan[c*x])^3/(e*(d + e*x)) - (3*b*c*(a + b*
ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/(c^2*d^2 + e^2) + (3*b*c*(a + b*ArcTan[c
*x])^2*Log[2/(1 + I*c*x)])/(c^2*d^2 + e^2) + (3*b*c*(a + b*ArcTan[c*x])^2*L
og[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(c^2*d^2 + e^2) + ((3*I)*b^2
*c*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/(c^2*d^2 + e^2) + ((3
*I)*b^2*c*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^2*d^2 + e^2
) - ((3*I)*b^2*c*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d +
 I*e)*(1 - I*c*x))])/(c^2*d^2 + e^2) - (3*b^3*c*PolyLog[3, 1 - 2/(1 - I*c*x
)])/(2*(c^2*d^2 + e^2)) + (3*b^3*c*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*(c^2*d
^2 + e^2)) + (3*b^3*c*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*
x))])/(2*(c^2*d^2 + e^2))
```

Rubi [A] time = 0.53011, antiderivative size = 499, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {4864, 4858, 4984, 4884, 4920, 4854, 4994, 6610}

$$\frac{3ib^2c\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^2d^2+e^2}+\frac{3ib^2c\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\left(a+b\tan^{-1}(cx)\right)}{c^2d^2+e^2}-\frac{3ib^2c\left(a+b\tan^{-1}(cx)\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^3/(d + e*x)^2,x]
```

```
[Out] (I*c*(a + b*ArcTan[c*x])^3)/(c^2*d^2 + e^2) + (c^2*d*(a + b*ArcTan[c*x])^3)
/(e*(c^2*d^2 + e^2)) - (a + b*ArcTan[c*x])^3/(e*(d + e*x)) - (3*b*c*(a + b*
ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/(c^2*d^2 + e^2) + (3*b*c*(a + b*ArcTan[c
*x])^2*Log[2/(1 + I*c*x)])/(c^2*d^2 + e^2) + (3*b*c*(a + b*ArcTan[c*x])^2*L
og[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(c^2*d^2 + e^2) + ((3*I)*b^2
*c*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/(c^2*d^2 + e^2) + ((3
*I)*b^2*c*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^2*d^2 + e^2
) - ((3*I)*b^2*c*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d +
```

```
 I*e)*(1 - I*c*x))])/(c^2*d^2 + e^2) - (3*b^3*c*PolyLog[3, 1 - 2/(1 - I*c*x
)])/(2*(c^2*d^2 + e^2)) + (3*b^3*c*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*(c^2*d
^2 + e^2)) + (3*b^3*c*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*
x))])/(2*(c^2*d^2 + e^2))
```

Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4858

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
-Simp[((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e, x] + (Simp[((a + b*ArcT
an[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x] + Simp[(I*
b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/e, x] - Simp[(I*b*(a +
 b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/
e, x] - Simp[(b^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e), x] + Simp[(b^2*Poly
Log[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e), x]) /; FreeQ[
{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4984

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{(d+ex)^2}\,dx = -\frac{\left(a+b\tan^{-1}(cx)\right)^3}{e(d+ex)} + \frac{(3bc)\int\left(\frac{e^2\left(a+b\tan^{-1}(cx)\right)^2}{\left(c^2d^2+e^2\right)(d+ex)} + \frac{c^2(d-ex)\left(a+b\tan^{-1}(cx)\right)^2}{\left(c^2d^2+e^2\right)\left(1+c^2x^2\right)}\right)dx}{e}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^3}{e(d+ex)} + \frac{\left(3bc^3\right)\int\frac{(d-ex)\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}\,dx}{e\left(c^2d^2+e^2\right)} + \frac{(3bce)\int\frac{\left(a+b\tan^{-1}(cx)\right)^2}{d+ex}\,dx}{c^2d^2+e^2}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^3}{e(d+ex)} - \frac{3bc\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1-icx}\right)}{c^2d^2+e^2} + \frac{3bc\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2c(}{(cd+i}\right.}{c^2d^2+e^2}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^3}{e(d+ex)} - \frac{3bc\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1-icx}\right)}{c^2d^2+e^2} + \frac{3bc\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2c(}{(cd+i}\right.}{c^2d^2+e^2}$$

$$= \frac{ic\left(a+b\tan^{-1}(cx)\right)^3}{c^2d^2+e^2} + \frac{c^2d\left(a+b\tan^{-1}(cx)\right)^3}{e\left(c^2d^2+e^2\right)} - \frac{\left(a+b\tan^{-1}(cx)\right)^3}{e(d+ex)} - \frac{3bc\left(a+b\tan^{-1}(cx)\right.}{c^2d^2+e^2}$$

$$= \frac{ic\left(a+b\tan^{-1}(cx)\right)^3}{c^2d^2+e^2} + \frac{c^2d\left(a+b\tan^{-1}(cx)\right)^3}{e\left(c^2d^2+e^2\right)} - \frac{\left(a+b\tan^{-1}(cx)\right)^3}{e(d+ex)} - \frac{3bc\left(a+b\tan^{-1}(cx)\right.}{c^2d^2+e^2}$$

$$= \frac{ic\left(a+b\tan^{-1}(cx)\right)^3}{c^2d^2+e^2} + \frac{c^2d\left(a+b\tan^{-1}(cx)\right)^3}{e\left(c^2d^2+e^2\right)} - \frac{\left(a+b\tan^{-1}(cx)\right)^3}{e(d+ex)} - \frac{3bc\left(a+b\tan^{-1}(cx)\right.}{c^2d^2+e^2}$$

$$= \frac{ic\left(a+b\tan^{-1}(cx)\right)^3}{c^2d^2+e^2} + \frac{c^2d\left(a+b\tan^{-1}(cx)\right)^3}{e\left(c^2d^2+e^2\right)} - \frac{\left(a+b\tan^{-1}(cx)\right)^3}{e(d+ex)} - \frac{3bc\left(a+b\tan^{-1}(cx)\right.}{c^2d^2+e^2}$$

Mathematica [F] time = 123.733, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{(d+ex)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x])^3/(d + e*x)^2,x]
```

```
[Out] Integrate[(a + b*ArcTan[c*x])^3/(d + e*x)^2, x]
```

Maple [C] time = 0.774, size = 2960, normalized size = 5.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^3/(e*x+d)^2,x)
```

```
[Out] -3/2*I*c*b^3/(c^2*d^2+e^2)*arctan(c*x)^2*Pi*csgn(I*(-I*(1+I*c*x)^2/(c^2*x^2
+1)*e+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+d*c))*csgn(I*(-I*(1+I*c*x)^2/(c^2*x^2
+1)*e+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+d*c)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2-3
*I*c^2*b^3/(c^2*d^2+e^2)*d/(d*c-I*e)*arctan(c*x)*polylog(2,(I*e-d*c)/(d*c+I
*e)*(1+I*c*x)^2/(c^2*x^2+1))-3*I*c*b^3*e*arctan(c*x)*polylog(2,(I*e-d*c)/(d
*c+I*e)*(1+I*c*x)^2/(c^2*x^2+1))/(c^2*d^2+e^2)/(e+I*d*c)-3/2*I*c*b^3/(c^2*d
^2+e^2)*arctan(c*x)^2*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*(-I*(1+
I*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+d*c)/((1+I*c*x)^2/(c
^2*x^2+1)+1))^2-3/4*I*c*b^3/(c^2*d^2+e^2)*arctan(c*x)^2*Pi*csgn(I*(1+I*c*x)
/(c^2*x^2+1)^(1/2))^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))+3/4*I*c*b^3/(c^2*d^2+
e^2)*arctan(c*x)^2*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c
^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2-3/2*I*c*b^3/(c^2*d^2+e^2)*arctan
(c*x)^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2
+1)+1)^2)^2+3/4*I*c*b^3/(c^2*d^2+e^2)*arctan(c*x)^2*Pi*csgn(I*((1+I*c*x)^2/
(c^2*x^2+1)+1))^2*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)+3/4*I*c*b^3/(c^2*d^
2+e^2)*arctan(c*x)^2*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c
*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^2+3/2*I*c*b^3/(c^2*d^2+e^2
)*arctan(c*x)^2*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*csgn(I*(1+I*c*x)^2/(
c^2*x^2+1))^2-3/2*c*b^3*arctan(c*x)^2/(c^2*d^2+e^2)*ln(c^2*x^2+1)-3*c*b^3/(
c^2*d^2+e^2)*arctan(c*x)^2*ln(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)^2/
(c^2*x^2+1)+I*e+d*c)+3*c*b^3/(c^2*d^2+e^2)*arctan(c*x)^2*ln((1+I*c*x)/(c^2*
x^2+1)^(1/2))-c*b^3/(c*e*x+c*d)/e*arctan(c*x)^3+3*c*b^3/(c^2*d^2+e^2)*arcta
n(c*x)^2*ln(2)-I*c*b^3/(c^2*d^2+e^2)*arctan(c*x)^3+3*c*a^2*b/(c^2*d^2+e^2)*
ln(c*e*x+c*d)-3/2*c*a^2*b/(c^2*d^2+e^2)*ln(c^2*x^2+1)+3/2*c^2*b^3/(c^2*d^2+
e^2)*d/(d*c-I*e)*polylog(3,(I*e-d*c)/(d*c+I*e)*(1+I*c*x)^2/(c^2*x^2+1))+c^2
*b^3/e*arctan(c*x)^3/(c^2*d^2+e^2)*d-3*c*a^2*b/(c*e*x+c*d)/e*arctan(c*x)-3*
c*a*b^2/(c*e*x+c*d)/e*arctan(c*x)^2+6*c*a*b^2*arctan(c*x)/(c^2*d^2+e^2)*ln(
c*e*x+c*d)-3*c*a*b^2*arctan(c*x)/(c^2*d^2+e^2)*ln(c^2*x^2+1)+3*I*c*a*b^2/(c
^2*d^2+e^2)*dilog((I*e-e*c*x)/(d*c+I*e))-3/4*I*c*a*b^2/(c^2*d^2+e^2)*ln(c*x
+I)^2+3/2*I*c*a*b^2/(c^2*d^2+e^2)*dilog(-1/2*I*(c*x+I))+3/4*I*c*a*b^2/(c^2*
d^2+e^2)*ln(c*x-I)^2-3/2*I*c*a*b^2/(c^2*d^2+e^2)*dilog(1/2*I*(c*x-I))-3*I*c
*a*b^2/(c^2*d^2+e^2)*dilog((I*e+e*c*x)/(I*e-d*c))+3/2*c*b^3*e*polylog(3,(I*
e-d*c)/(d*c+I*e)*(1+I*c*x)^2/(c^2*x^2+1))/(c^2*d^2+e^2)/(e+I*d*c)+3/4*I*c*b
^3/(c^2*d^2+e^2)*arctan(c*x)^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)+1)^2)^3+3
/2*I*c*a*b^2/(c^2*d^2+e^2)*ln(c*x-I)*ln(-1/2*I*(c*x+I))+3/2*I*c*a*b^2/(c^2*
d^2+e^2)*ln(c^2*x^2+1)*ln(c*x+I)-3/2*I*c*a*b^2/(c^2*d^2+e^2)*ln(c*x+I)*ln(1
```

```
/2*I*(c*x-I))+3*I*c*a*b^2/(c^2*d^2+e^2)*ln(c*e*x+c*d)*ln((I*e-e*c*x)/(d*c+I
*e))-c*a^3/(c*e*x+c*d)/e-3*I*c*a*b^2/(c^2*d^2+e^2)*ln(c*e*x+c*d)*ln((I*e+e*
c*x)/(I*e-d*c))-3/2*I*c*a*b^2/(c^2*d^2+e^2)*ln(c^2*x^2+1)*ln(c*x-I)-3/4*I*c
*b^3/(c^2*d^2+e^2)*arctan(c*x)^2*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*
x)^2/(c^2*x^2+1)+1)^2)^3+3*c^2*a^2*b/e/(c^2*d^2+e^2)*d*arctan(c*x)+3*c^2*a*
b^2/e/(c^2*d^2+e^2)*d*arctan(c*x)^2+3*c*b^3*e*arctan(c*x)^2*ln(1-(I*e-d*c)/
(d*c+I*e)*(1+I*c*x)^2/(c^2*x^2+1))/(c^2*d^2+e^2)/(e+I*d*c)+3*c^2*b^3/(c^2*d
^2+e^2)*d/(d*c-I*e)*arctan(c*x)^2*ln(1-(I*e-d*c)/(d*c+I*e)*(1+I*c*x)^2/(c^2
*x^2+1))-3/4*I*c*b^3/(c^2*d^2+e^2)*arctan(c*x)^2*Pi*csgn(I*(1+I*c*x)^2/(c^2
*x^2+1))^3+3/2*I*c*b^3/(c^2*d^2+e^2)*arctan(c*x)^2*Pi*csgn(I*(-I*(1+I*c*x)^
2/(c^2*x^2+1)*e+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+d*c)/((1+I*c*x)^2/(c^2*x^2+
1)+1))^3+3/2*I*c*b^3/(c^2*d^2+e^2)*arctan(c*x)^2*Pi*csgn(I/((1+I*c*x)^2/(c^
2*x^2+1)+1))*csgn(I*(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)^2/(c^2*x^2+
1)+I*e+d*c))*csgn(I*(-I*(1+I*c*x)^2/(c^2*x^2+1)*e+c*d*(1+I*c*x)^2/(c^2*x^2+
1)+I*e+d*c)/((1+I*c*x)^2/(c^2*x^2+1)+1))-3/4*I*c*b^3/(c^2*d^2+e^2)*arctan(c
*x)^2*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+
1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/((1+I*c*x)^2/(c^2*x^2+1)+1)^2)+3*c*b^3*a
rctan(c*x)^2/(c^2*d^2+e^2)*ln(c*e*x+c*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3}{2}\left(\left(\frac{2\,cd\arctan\left(cx\right)}{c^2d^2e+e^3}-\frac{\log\left(c^2x^2+1\right)}{c^2d^2+e^2}+\frac{2\,\log\left(ex+d\right)}{c^2d^2+e^2}\right)c-\frac{2\,\arctan\left(cx\right)}{e^2x+de}\right)a^2b-\frac{a^3}{e^2x+de}-\frac{\frac{15}{2}\,b^3\arctan\left(cx\right)^3-\frac{21}{8}\,b^3\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 3/2*((2*c*d*arctan(c*x)/(c^2*d^2*e + e^3) - log(c^2*x^2 + 1)/(c^2*d^2 + e^2
) + 2*log(e*x + d)/(c^2*d^2 + e^2))*c - 2*arctan(c*x)/(e^2*x + d*e))*a^2*b
- a^3/(e^2*x + d*e) - 1/32*(4*b^3*arctan(c*x)^3 - 3*b^3*arctan(c*x)*log(c^2
*x^2 + 1)^2 - 32*(e^2*x + d*e)*integrate(1/32*(28*(b^3*c^2*e*x^2 + b^3*e)*a
rctan(c*x)^3 + 12*(8*a*b^2*c^2*e*x^2 + b^3*c*e*x + b^3*c*d + 8*a*b^2*e)*arc
tan(c*x)^2 - 12*(b^3*c^2*e*x^2 + b^3*c^2*d*x)*arctan(c*x)*log(c^2*x^2 + 1)
- 3*(b^3*c*e*x + b^3*c*d - (b^3*c^2*e*x^2 + b^3*e)*arctan(c*x))*log(c^2*x^2
 + 1)^2)/(c^2*e^3*x^4 + 2*c^2*d*e^2*x^3 + 2*d*e^2*x + d^2*e + (c^2*d^2*e +
e^3)*x^2), x))/(e^2*x + d*e)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3\arctan(cx)^3 + 3ab^2\arctan(cx)^2 + 3a^2b\arctan(cx) + a^3}{e^2x^2 + 2dex + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x) +
 a^3)/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{atan}(cx))^3}{(d + ex)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**3/(e*x+d)**2,x)
```

```
[Out] Integral((a + b*atan(c*x))**3/(d + e*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\arctan(cx) + a)^3}{(ex + d)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^3/(e*x + d)^2, x)
```

3.20 $\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{(d+ex)^3}\,dx$

Optimal. Leaf size=936

result too large to display

```
[Out] (3*b*c^3*d*(a + b*ArcTan[c*x])^2)/(2*(c^2*d^2 + e^2)^2) + (((3*I)/2)*b*c^2*
e*(a + b*ArcTan[c*x])^2)/(c^2*d^2 + e^2)^2 - (3*b*c*(a + b*ArcTan[c*x])^2)/
(2*(c^2*d^2 + e^2)*(d + e*x)) + (I*c^3*d*(a + b*ArcTan[c*x])^3)/(c^2*d^2 +
e^2)^2 + (c^2*(c*d - e)*(c*d + e)*(a + b*ArcTan[c*x])^3)/(2*e*(c^2*d^2 + e^
2)^2) - (a + b*ArcTan[c*x])^3/(2*e*(d + e*x)^2) - (3*b^2*c^2*e*(a + b*ArcTa
n[c*x])*Log[2/(1 - I*c*x)])/(c^2*d^2 + e^2)^2 - (3*b*c^3*d*(a + b*ArcTan[c*
x])^2*Log[2/(1 - I*c*x)])/(c^2*d^2 + e^2)^2 + (3*b^2*c^2*e*(a + b*ArcTan[c*
x])*Log[2/(1 + I*c*x)])/(c^2*d^2 + e^2)^2 + (3*b*c^3*d*(a + b*ArcTan[c*x])^
2*Log[2/(1 + I*c*x)])/(c^2*d^2 + e^2)^2 + (3*b^2*c^2*e*(a + b*ArcTan[c*x])*
Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(c^2*d^2 + e^2)^2 + (3*b*c^
3*d*(a + b*ArcTan[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(
c^2*d^2 + e^2)^2 + (((3*I)/2)*b^3*c^2*e*PolyLog[2, 1 - 2/(1 - I*c*x)])/(c^2
*d^2 + e^2)^2 + ((3*I)*b^2*c^3*d*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 -
I*c*x)])/(c^2*d^2 + e^2)^2 + (((3*I)/2)*b^3*c^2*e*PolyLog[2, 1 - 2/(1 + I*c
*x)])/(c^2*d^2 + e^2)^2 + ((3*I)*b^2*c^3*d*(a + b*ArcTan[c*x])*PolyLog[2, 1
 - 2/(1 + I*c*x)])/(c^2*d^2 + e^2)^2 - (((3*I)/2)*b^3*c^2*e*PolyLog[2, 1 -
(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(c^2*d^2 + e^2)^2 - ((3*I)*b^2*
c^3*d*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 -
I*c*x))])/(c^2*d^2 + e^2)^2 - (3*b^3*c^3*d*PolyLog[3, 1 - 2/(1 - I*c*x)])/(
2*(c^2*d^2 + e^2)^2) + (3*b^3*c^3*d*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*(c^2*
d^2 + e^2)^2) + (3*b^3*c^3*d*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1
 - I*c*x))])/(2*(c^2*d^2 + e^2)^2)
```

Rubi [A] time = 1.08992, antiderivative size = 936, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4864, 4856, 2402, 2315, 2447, 4984, 4884, 4920, 4854, 4858, 4994, 6610}

$$\frac{3ic^2e\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)b^3}{2\left(c^2d^2+e^2\right)^2}+\frac{3ic^2e\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)b^3}{2\left(c^2d^2+e^2\right)^2}-\frac{3ic^2e\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)b^3}{2\left(c^2d^2+e^2\right)^2}-\frac{3c^3d\,\text{PolyLo}}{2\left(c^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^3/(d + e*x)^3,x]
```

```
[Out] (3*b*c^3*d*(a + b*ArcTan[c*x])^2)/(2*(c^2*d^2 + e^2)^2) + (((3*I)/2)*b*c^2*
e*(a + b*ArcTan[c*x])^2)/(c^2*d^2 + e^2)^2 - (3*b*c*(a + b*ArcTan[c*x])^2)/
(2*(c^2*d^2 + e^2)*(d + e*x)) + (I*c^3*d*(a + b*ArcTan[c*x])^3)/(c^2*d^2 +
e^2)^2 + (c^2*(c*d - e)*(c*d + e)*(a + b*ArcTan[c*x])^3)/(2*e*(c^2*d^2 + e^
2)^2) - (a + b*ArcTan[c*x])^3/(2*e*(d + e*x)^2) - (3*b^2*c^2*e*(a + b*ArcTa
n[c*x])*Log[2/(1 - I*c*x)])/(c^2*d^2 + e^2)^2 - (3*b*c^3*d*(a + b*ArcTan[c*
x])^2*Log[2/(1 - I*c*x)])/(c^2*d^2 + e^2)^2 + (3*b^2*c^2*e*(a + b*ArcTan[c*
x])*Log[2/(1 + I*c*x)])/(c^2*d^2 + e^2)^2 + (3*b*c^3*d*(a + b*ArcTan[c*x])^
2*Log[2/(1 + I*c*x)])/(c^2*d^2 + e^2)^2 + (3*b^2*c^2*e*(a + b*ArcTan[c*x])*
Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(c^2*d^2 + e^2)^2 + (3*b*c^
3*d*(a + b*ArcTan[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(
c^2*d^2 + e^2)^2 + (((3*I)/2)*b^3*c^2*e*PolyLog[2, 1 - 2/(1 - I*c*x)])/(c^2
*d^2 + e^2)^2 + ((3*I)*b^2*c^3*d*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 -
I*c*x)])/(c^2*d^2 + e^2)^2 + (((3*I)/2)*b^3*c^2*e*PolyLog[2, 1 - 2/(1 + I*c
*x)])/(c^2*d^2 + e^2)^2 + ((3*I)*b^2*c^3*d*(a + b*ArcTan[c*x])*PolyLog[2, 1
 - 2/(1 + I*c*x)])/(c^2*d^2 + e^2)^2 - (((3*I)/2)*b^3*c^2*e*PolyLog[2, 1 -
(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(c^2*d^2 + e^2)^2 - ((3*I)*b^2*
c^3*d*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 -
I*c*x))])/(c^2*d^2 + e^2)^2 - (3*b^3*c^3*d*PolyLog[3, 1 - 2/(1 - I*c*x)])/(
2*(c^2*d^2 + e^2)^2) + (3*b^3*c^3*d*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*(c^2*
d^2 + e^2)^2) + (3*b^3*c^3*d*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1
 - I*c*x))])/(2*(c^2*d^2 + e^2)^2)
```

Rule 4864

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x])^p)/(e*(q + 1)), x] - D
ist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1), (
d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4984

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/((
d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p
/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGt
Q[p, 0] && EqQ[e, c^2*d] && IGtQ[m, 0]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4858

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
-Simp[((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e, x] + (Simp[((a + b*ArcT
an[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x] + Simp[(I*
```

```
b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/e, x] - Simp[(I*b*(a +
 b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/
e, x] - Simp[(b^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e), x] + Simp[(b^2*Poly
Log[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e), x]) /; FreeQ[
{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4994

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p*I)/2, Int[((a + b*ArcTan[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
 + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - (2*I)/(I - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{(d+ex)^3}\,dx = -\frac{\left(a+b\tan^{-1}(cx)\right)^3}{2e(d+ex)^2} + \frac{(3bc)\int\left(\frac{e^2\left(a+b\tan^{-1}(cx)\right)^2}{\left(c^2d^2+e^2\right)(d+ex)^2} + \frac{2c^2de^2\left(a+b\tan^{-1}(cx)\right)^2}{\left(c^2d^2+e^2\right)^2(d+ex)} + \frac{\left(c^4d^2-c^2e^2-2c^4dex\right)\left(a+b\,\mathrm{t}\right.}{\left(c^2d^2+e^2\right)^2\left(1+c^2x\right.}}{2e}$$

$$= -\frac{\left(a+b\tan^{-1}(cx)\right)^3}{2e(d+ex)^2} + \frac{(3bc)\int\frac{\left(c^4d^2-c^2e^2-2c^4dex\right)\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}\,dx}{2e\left(c^2d^2+e^2\right)^2} + \frac{\left(3bc^3de\right)\int\frac{\left(a+b\tan^{-1}(cx)\right)^2}{d+ex}\,d}{\left(c^2d^2+e^2\right)^2}$$

$$= -\frac{3bc\left(a+b\tan^{-1}(cx)\right)^2}{2\left(c^2d^2+e^2\right)(d+ex)} - \frac{\left(a+b\tan^{-1}(cx)\right)^3}{2e(d+ex)^2} - \frac{3bc^3d\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1-icx}\right)}{\left(c^2d^2+e^2\right)^2} + \frac{3bc^3}{}$$

$$= -\frac{3bc\left(a+b\tan^{-1}(cx)\right)^2}{2\left(c^2d^2+e^2\right)(d+ex)} - \frac{\left(a+b\tan^{-1}(cx)\right)^3}{2e(d+ex)^2} - \frac{3bc^3d\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1-icx}\right)}{\left(c^2d^2+e^2\right)^2} + \frac{3bc^3}{}$$

$$= -\frac{3bc\left(a+b\tan^{-1}(cx)\right)^2}{2\left(c^2d^2+e^2\right)(d+ex)} + \frac{ic^3d\left(a+b\tan^{-1}(cx)\right)^3}{\left(c^2d^2+e^2\right)^2} + \frac{c^2(cd-e)(cd+e)\left(a+b\tan^{-1}(cx)\right)^3}{2e\left(c^2d^2+e^2\right)^2} -$$

$$= -\frac{3bc\left(a+b\tan^{-1}(cx)\right)^2}{2\left(c^2d^2+e^2\right)(d+ex)} + \frac{ic^3d\left(a+b\tan^{-1}(cx)\right)^3}{\left(c^2d^2+e^2\right)^2} + \frac{c^2(cd-e)(cd+e)\left(a+b\tan^{-1}(cx)\right)^3}{2e\left(c^2d^2+e^2\right)^2} -$$

$$= \frac{3bc^3d\left(a+b\tan^{-1}(cx)\right)^2}{2\left(c^2d^2+e^2\right)^2} + \frac{3ibc^2e\left(a+b\tan^{-1}(cx)\right)^2}{2\left(c^2d^2+e^2\right)^2} - \frac{3bc\left(a+b\tan^{-1}(cx)\right)^2}{2\left(c^2d^2+e^2\right)(d+ex)} + \frac{ic^3d\left(a+b\right.}{\left(c^2d\right.}$$

$$= \frac{3bc^3d\left(a+b\tan^{-1}(cx)\right)^2}{2\left(c^2d^2+e^2\right)^2} + \frac{3ibc^2e\left(a+b\tan^{-1}(cx)\right)^2}{2\left(c^2d^2+e^2\right)^2} - \frac{3bc\left(a+b\tan^{-1}(cx)\right)^2}{2\left(c^2d^2+e^2\right)(d+ex)} + \frac{ic^3d\left(a+b\right.}{\left(c^2d\right.}$$

$$= \frac{3bc^3d\left(a+b\tan^{-1}(cx)\right)^2}{2\left(c^2d^2+e^2\right)^2} + \frac{3ibc^2e\left(a+b\tan^{-1}(cx)\right)^2}{2\left(c^2d^2+e^2\right)^2} - \frac{3bc\left(a+b\tan^{-1}(cx)\right)^2}{2\left(c^2d^2+e^2\right)(d+ex)} + \frac{ic^3d\left(a+b\right.}{\left(c^2d\right.}$$

$$= \frac{3bc^3d\left(a+b\tan^{-1}(cx)\right)^2}{2\left(c^2d^2+e^2\right)^2} + \frac{3ibc^2e\left(a+b\tan^{-1}(cx)\right)^2}{2\left(c^2d^2+e^2\right)^2} - \frac{3bc\left(a+b\tan^{-1}(cx)\right)^2}{2\left(c^2d^2+e^2\right)(d+ex)} + \frac{ic^3d\left(a+b\right.}{\left(c^2d\right.}$$

Mathematica [F] time = 67.1242, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{(d+ex)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x])^3/(d + e*x)^3,x]

[Out] Integrate[(a + b*ArcTan[c*x])^3/(d + e*x)^3, x]
```

Maple [C] time = 6.972, size = 41013, normalized size = 43.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^3/(e*x+d)^3,x)

[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/(e*x+d)^3,x, algorithm="maxima")

[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3 \arctan(cx)^3 + 3ab^2 \arctan(cx)^2 + 3a^2 b \arctan(cx) + a^3}{e^3x^3 + 3de^2x^2 + 3d^2ex + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x) +
 a^3)/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**3/(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \arctan(cx) + a)^3}{(ex + d)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x) + a)^3/(e*x + d)^3, x)
```

3.21 $\int (d+ex)^2 \left(a+b\tan^{-1}\left(cx^2\right)\right) dx$

Optimal. Leaf size=250

$$\frac{(d+ex)^3\left(a+b\tan^{-1}\left(cx^2\right)\right)}{3e} - \frac{b\left(3cd^2+e^2\right)\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{6\sqrt{2}c^{3/2}} + \frac{b\left(3cd^2+e^2\right)\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{6\sqrt{2}c^{3/2}} + \frac{b\left(3cd^2-e\right.}{}$$

```
[Out] (-2*b*e^2*x)/(3*c) - (b*d^3*ArcTan[c*x^2])/(3*e) + ((d + e*x)^3*(a + b*ArcT
an[c*x^2]))/(3*e) + (b*(3*c*d^2 - e^2)*ArcTan[1 - Sqrt[2]*Sqrt[c]*x])/(3*Sq
rt[2]*c^(3/2)) - (b*(3*c*d^2 - e^2)*ArcTan[1 + Sqrt[2]*Sqrt[c]*x])/(3*Sqrt[
2]*c^(3/2)) - (b*(3*c*d^2 + e^2)*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2])/(6*Sqr
t[2]*c^(3/2)) + (b*(3*c*d^2 + e^2)*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(6*S
qrt[2]*c^(3/2)) - (b*d*e*Log[1 + c^2*x^4])/(2*c)
```

Rubi [A] time = 0.303477, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 14, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {5205, 12, 1831, 1248, 635, 203, 260, 1280, 1168, 1162, 617, 204, 1165, 628}

$$\frac{(d+ex)^3\left(a+b\tan^{-1}\left(cx^2\right)\right)}{3e} - \frac{b\left(3cd^2+e^2\right)\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{6\sqrt{2}c^{3/2}} + \frac{b\left(3cd^2+e^2\right)\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{6\sqrt{2}c^{3/2}} + \frac{b\left(3cd^2-e\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] (-2*b*e^2*x)/(3*c) - (b*d^3*ArcTan[c*x^2])/(3*e) + ((d + e*x)^3*(a + b*ArcT
an[c*x^2]))/(3*e) + (b*(3*c*d^2 - e^2)*ArcTan[1 - Sqrt[2]*Sqrt[c]*x])/(3*Sq
rt[2]*c^(3/2)) - (b*(3*c*d^2 - e^2)*ArcTan[1 + Sqrt[2]*Sqrt[c]*x])/(3*Sqrt[
2]*c^(3/2)) - (b*(3*c*d^2 + e^2)*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2])/(6*Sqr
t[2]*c^(3/2)) + (b*(3*c*d^2 + e^2)*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(6*S
qrt[2]*c^(3/2)) - (b*d*e*Log[1 + c^2*x^4])/(2*c)
```

Rule 5205

```
Int[((a_.) + ArcTan[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((c + d*x)^(m + 1)*(a + b*ArcTan[u]))/(d*(m + 1)), x] - Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 + u^2), x], x], x]
/; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&
  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m +
1, x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1831

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[((c*x)^(m + ii)*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
))/(c^ii*(a + b*x^n)), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1280

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int (d+ex)^2\left(a+b\tan^{-1}\left(cx^2\right)\right)dx = \frac{(d+ex)^3\left(a+b\tan^{-1}\left(cx^2\right)\right)}{3e} - \frac{b\int \frac{2cx(d+ex)^3}{1+c^2x^4}\,dx}{3e}$$

$$= \frac{(d+ex)^3\left(a+b\tan^{-1}\left(cx^2\right)\right)}{3e} - \frac{(2bc)\int \frac{x(d+ex)^3}{1+c^2x^4}\,dx}{3e}$$

$$= \frac{(d+ex)^3\left(a+b\tan^{-1}\left(cx^2\right)\right)}{3e} - \frac{(2bc)\int \left(\frac{x\left(d^3+3de^2x^2\right)}{1+c^2x^4} + \frac{x^2\left(3d^2e+e^3x^2\right)}{1+c^2x^4}\right)dx}{3e}$$

$$= \frac{(d+ex)^3\left(a+b\tan^{-1}\left(cx^2\right)\right)}{3e} - \frac{(2bc)\int \frac{x\left(d^3+3de^2x^2\right)}{1+c^2x^4}\,dx}{3e} - \frac{(2bc)\int \frac{x^2\left(3d^2e+e^3x^2\right)}{1+c^2x^4}\,dx}{3e}$$

$$= -\frac{2be^2x}{3c} + \frac{(d+ex)^3\left(a+b\tan^{-1}\left(cx^2\right)\right)}{3e} + \frac{(2b)\int \frac{e^3-3c^2d^2ex^2}{1+c^2x^4}\,dx}{3ce} - \frac{(bc)\,\mathrm{Subst}\left(\int \frac{d^3+}{1+}\right.}{3e}$$

$$= -\frac{2be^2x}{3c} + \frac{(d+ex)^3\left(a+b\tan^{-1}\left(cx^2\right)\right)}{3e} - \frac{\left(bcd^3\right)\mathrm{Subst}\left(\int \frac{1}{1+c^2x^2}\,dx, x, x^2\right)}{3e} - (bcd$$

$$= -\frac{2be^2x}{3c} - \frac{bd^3\tan^{-1}\left(cx^2\right)}{3e} + \frac{(d+ex)^3\left(a+b\tan^{-1}\left(cx^2\right)\right)}{3e} - \frac{bde\log\left(1+c^2x^4\right)}{2c} - -$$

$$= -\frac{2be^2x}{3c} - \frac{bd^3\tan^{-1}\left(cx^2\right)}{3e} + \frac{(d+ex)^3\left(a+b\tan^{-1}\left(cx^2\right)\right)}{3e} - \frac{b\left(3cd^2+e^2\right)\log\left(1-\right.}{6\sqrt{2}c^{3/}}$$

$$= -\frac{2be^2x}{3c} - \frac{bd^3\tan^{-1}\left(cx^2\right)}{3e} + \frac{(d+ex)^3\left(a+b\tan^{-1}\left(cx^2\right)\right)}{3e} + \frac{b\left(3cd^2-e^2\right)\tan^{-1}\left(1\right.}{3\sqrt{2}c^{3/2}}$$

Mathematica [A] time = 3.24062, size = 252, normalized size = 1.01

$$\frac{1}{12}\left(12ad^2x + 12adex^2 + 4ae^2x^3 - \frac{\sqrt{2}b\left(3cd^2+e^2\right)\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{c^{3/2}} + \frac{\sqrt{2}b\left(3cd^2+e^2\right)\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{c^{3/2}} + \right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] (12*a*d^2*x - (8*b*e^2*x)/c + 12*a*d*e*x^2 + 4*a*e^2*x^3 + 4*b*x*(3*d^2 + 3
*d*e*x + e^2*x^2)*ArcTan[c*x^2] + (2*Sqrt[2]*b*(3*c*d^2 - e^2)*ArcTan[1 - S
qrt[2]*Sqrt[c]*x])/c^(3/2) - (2*Sqrt[2]*b*(3*c*d^2 - e^2)*ArcTan[1 + Sqrt[2
]*Sqrt[c]*x])/c^(3/2) - (Sqrt[2]*b*(3*c*d^2 + e^2)*Log[1 - Sqrt[2]*Sqrt[c]*
x + c*x^2])/c^(3/2) + (Sqrt[2]*b*(3*c*d^2 + e^2)*Log[1 + Sqrt[2]*Sqrt[c]*x
```

```
+ c*x^2])/c^(3/2) - (6*b*d*e*Log[1 + c^2*x^4])/c)/12
```

Maple [A] time = 0.033, size = 381, normalized size = 1.5

$$\frac{ae^2x^3}{3} + aex^2d + axd^2 + \frac{ad^3}{3e} + \frac{be^2\arctan\left(cx^2\right)x^3}{3} + be\arctan\left(cx^2\right)x^2d + b\arctan\left(cx^2\right)xd^2 + \frac{bd^3\arctan\left(cx^2\right)}{3e} - \frac{2be}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(a+b*arctan(c*x^2)),x)
```

```
[Out] 1/3*a*e^2*x^3+a*e*x^2*d+a*x*d^2+1/3*a/e*d^3+1/3*b*e^2*arctan(c*x^2)*x^3+b*e
*arctan(c*x^2)*x^2*d+b*arctan(c*x^2)*x*d^2+1/3*b*d^3*arctan(c*x^2)/e-2/3*b*
e^2*x/c+1/6*b*e^2/c*(1/c^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c^2)^(1/4)*x+1)
+1/6*b*e^2/c*(1/c^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c^2)^(1/4)*x-1)+1/12*b
*e^2/c*(1/c^2)^(1/4)*2^(1/2)*ln((x^2+(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2))
/(x^2-(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2)))-1/3*b/e*c*d^3/(c^2)^(1/2)*arc
tan(x^2*(c^2)^(1/2))-1/4*b/c*d^2/(1/c^2)^(1/4)*2^(1/2)*ln((x^2-(1/c^2)^(1/4
)*x*2^(1/2)+(1/c^2)^(1/2))/(x^2+(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2)))-1/2
*b/c*d^2/(1/c^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c^2)^(1/4)*x+1)-1/2*b/c*d^
2/(1/c^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c^2)^(1/4)*x-1)-1/2*b*d*e*ln(c^2*
x^4+1)/c
```

Maxima [B] time = 1.5398, size = 771, normalized size = 3.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/3*a*e^2*x^3 + a*d*e*x^2 + 1/4*(c*(sqrt(2)*log(sqrt(c^2)*x^2 + sqrt(2)*(c^
2)^(1/4)*x + 1)/(c^2)^(3/4) - sqrt(2)*log(sqrt(c^2)*x^2 - sqrt(2)*(c^2)^(1/
4)*x + 1)/(c^2)^(3/4) - sqrt(2)*log((2*sqrt(c^2)*x - sqrt(2)*sqrt(-sqrt(c^2
)) + sqrt(2)*(c^2)^(1/4))/(2*sqrt(c^2)*x + sqrt(2)*sqrt(-sqrt(c^2)) + sqrt(
2)*(c^2)^(1/4)))/(sqrt(c^2)*sqrt(-sqrt(c^2))) - sqrt(2)*log((2*sqrt(c^2)*x
- sqrt(2)*sqrt(-sqrt(c^2)) - sqrt(2)*(c^2)^(1/4))/(2*sqrt(c^2)*x + sqrt(2)*
sqrt(-sqrt(c^2)) - sqrt(2)*(c^2)^(1/4)))/(sqrt(c^2)*sqrt(-sqrt(c^2)))) + 4*
x*arctan(c*x^2))*b*d^2 + 1/12*(4*x^3*arctan(c*x^2) + c*((sqrt(2)*log(sqrt(c
```

```
^2)*x^2 + sqrt(2)*(c^2)^(1/4)*x + 1)/(c^2)^(1/4) - sqrt(2)*log(sqrt(c^2)*x^
2 - sqrt(2)*(c^2)^(1/4)*x + 1)/(c^2)^(1/4) + sqrt(2)*log((2*sqrt(c^2)*x - s
qrt(2)*sqrt(-sqrt(c^2)) + sqrt(2)*(c^2)^(1/4))/(2*sqrt(c^2)*x + sqrt(2)*sqr
t(-sqrt(c^2)) + sqrt(2)*(c^2)^(1/4)))/sqrt(-sqrt(c^2)) + sqrt(2)*log((2*sqr
t(c^2)*x - sqrt(2)*sqrt(-sqrt(c^2)) - sqrt(2)*(c^2)^(1/4))/(2*sqrt(c^2)*x +
 sqrt(2)*sqrt(-sqrt(c^2)) - sqrt(2)*(c^2)^(1/4)))/sqrt(-sqrt(c^2)))/c^2 - 8
*x/c^2))*b*e^2 + a*d^2*x + 1/2*(2*c*x^2*arctan(c*x^2) - log(c^2*x^4 + 1))*b
*d*e/c
```

Fricas [B] time = 4.64603, size = 9956, normalized size = 39.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arctan(c*x^2)),x, algorithm="fricas")
```

```
[Out] -1/12*(4*sqrt(2)*c^7*sqrt((81*b^2*c^4*d^8 + 18*b^2*c^2*d^4*e^4 + b^2*e^8 -
6*c^4*d^2*e^2*sqrt((81*b^4*c^4*d^8 + 18*b^4*c^2*d^4*e^4 + b^4*e^8)/c^6))/(8
1*b^2*c^4*d^8 - 18*b^2*c^2*d^4*e^4 + b^2*e^8))*((81*b^4*c^4*d^8 + 18*b^4*c^
2*d^4*e^4 + b^4*e^8)/c^6)^(3/4)*sqrt((81*b^4*c^4*d^8 - 18*b^4*c^2*d^4*e^4 +
 b^4*e^8)/c^6)*arctan((sqrt(2)*(c^11*e^2*sqrt((81*b^4*c^4*d^8 + 18*b^4*c^2*
d^4*e^4 + b^4*e^8)/c^6)*sqrt((81*b^4*c^4*d^8 - 18*b^4*c^2*d^4*e^4 + b^4*e^8
)/c^6) + 3*(9*b^2*c^11*d^6 + b^2*c^9*d^2*e^4)*sqrt((81*b^4*c^4*d^8 - 18*b^4
*c^2*d^4*e^4 + b^4*e^8)/c^6))*sqrt((6561*b^6*c^8*d^16 - 162*b^6*c^4*d^8*e^8
 + b^6*e^16)*x^2 + sqrt(2)*(3*(81*b^3*c^9*d^10 - 18*b^3*c^7*d^6*e^4 + b^3*c
^5*d^2*e^8)*x*sqrt((81*b^4*c^4*d^8 + 18*b^4*c^2*d^4*e^4 + b^4*e^8)/c^6) + (
729*b^5*c^7*d^12*e^2 - 81*b^5*c^5*d^8*e^6 - 9*b^5*c^3*d^4*e^10 + b^5*c*e^14
)*x)*sqrt((81*b^2*c^4*d^8 + 18*b^2*c^2*d^4*e^4 + b^2*e^8 - 6*c^4*d^2*e^2*sq
rt((81*b^4*c^4*d^8 + 18*b^4*c^2*d^4*e^4 + b^4*e^8)/c^6))/(81*b^2*c^4*d^8 -
18*b^2*c^2*d^4*e^4 + b^2*e^8))*((81*b^4*c^4*d^8 + 18*b^4*c^2*d^4*e^4 + b^4*
e^8)/c^6)^(1/4) + (729*b^4*c^8*d^12 - 81*b^4*c^6*d^8*e^4 - 9*b^4*c^4*d^4*e^
8 + b^4*c^2*e^12)*sqrt((81*b^4*c^4*d^8 + 18*b^4*c^2*d^4*e^4 + b^4*e^8)/c^6)
)*sqrt((81*b^2*c^4*d^8 + 18*b^2*c^2*d^4*e^4 + b^2*e^8 - 6*c^4*d^2*e^2*sqrt(
(81*b^4*c^4*d^8 + 18*b^4*c^2*d^4*e^4 + b^4*e^8)/c^6))/(81*b^2*c^4*d^8 - 18*
b^2*c^2*d^4*e^4 + b^2*e^8))*((81*b^4*c^4*d^8 + 18*b^4*c^2*d^4*e^4 + b^4*e^8
)/c^6)^(3/4) + sqrt(2)*((81*b^3*c^15*d^8*e^2 - b^3*c^11*e^10)*x*sqrt((81*b^
4*c^4*d^8 + 18*b^4*c^2*d^4*e^4 + b^4*e^8)/c^6)*sqrt((81*b^4*c^4*d^8 - 18*b^
4*c^2*d^4*e^4 + b^4*e^8)/c^6) + 3*(729*b^5*c^15*d^14 + 81*b^5*c^13*d^10*e^4
 - 9*b^5*c^11*d^6*e^8 - b^5*c^9*d^2*e^12)*x*sqrt((81*b^4*c^4*d^8 - 18*b^4*c
^2*d^4*e^4 + b^4*e^8)/c^6))*sqrt((81*b^2*c^4*d^8 + 18*b^2*c^2*d^4*e^4 + b^2
*e^8 - 6*c^4*d^2*e^2*sqrt((81*b^4*c^4*d^8 + 18*b^4*c^2*d^4*e^4 + b^4*e^8)/c
```

```
^6))/(81*b^2*c^4*d^8 - 18*b^2*c^2*d^4*e^4 + b^2*e^8))*((81*b^4*c^4*d^8 + 18
*b^4*c^2*d^4*e^4 + b^4*e^8)/c^6)^(3/4) + (6561*b^6*c^14*d^16 + 1458*b^6*c^1
2*d^12*e^4 - 18*b^6*c^8*d^4*e^12 - b^6*c^6*e^16)*sqrt((81*b^4*c^4*d^8 + 18*
b^4*c^2*d^4*e^4 + b^4*e^8)/c^6)*sqrt((81*b^4*c^4*d^8 - 18*b^4*c^2*d^4*e^4 +
 b^4*e^8)/c^6))/(531441*b^10*c^12*d^24 + 118098*b^10*c^10*d^20*e^4 - 6561*b
^10*c^8*d^16*e^8 - 2916*b^10*c^6*d^12*e^12 - 81*b^10*c^4*d^8*e^16 + 18*b^10
*c^2*d^4*e^20 + b^10*e^24)) + 4*sqrt(2)*c^7*sqrt((81*b^2*c^4*d^8 + 18*b^2*c
^2*d^4*e^4 + b^2*e^8 - 6*c^4*d^2*e^2*sqrt((81*b^4*c^4*d^8 + 18*b^4*c^2*d^4*
e^4 + b^4*e^8)/c^6))/(81*b^2*c^4*d^8 - 18*b^2*c^2*d^4*e^4 + b^2*e^8))*((81*
b^4*c^4*d^8 + 18*b^4*c^2*d^4*e^4 + b^4*e^8)/c^6)^(3/4)*sqrt((81*b^4*c^4*d^8
 - 18*b^4*c^2*d^4*e^4 + b^4*e^8)/c^6)*arctan((sqrt(2)*(c^11*e^2*sqrt((81*b^
4*c^4*d^8 + 18*b^4*c^2*d^4*e^4 + b^4*e^8)/c^6)*sqrt((81*b^4*c^4*d^8 - 18*b^
4*c^2*d^4*e^4 + b^4*e^8)/c^6) + 3*(9*b^2*c^11*d^6 + b^2*c^9*d^2*e^4)*sqrt((
81*b^4*c^4*d^8 - 18*b^4*c^2*d^4*e^4 + b^4*e^8)/c^6))*sqrt((6561*b^6*c^8*d^1
6 - 162*b^6*c^4*d^8*e^8 + b^6*e^16)*x^2 - sqrt(2)*(3*(81*b^3*c^9*d^10 - 18*
b^3*c^7*d^6*e^4 + b^3*c^5*d^2*e^8)*x*sqrt((81*b^4*c^4*d^8 + 18*b^4*c^2*d^4*
e^4 + b^4*e^8)/c^6) + (729*b^5*c^7*d^12*e^2 - 81*b^5*c^5*d^8*e^6 - 9*b^5*c^
3*d^4*e^10 + b^5*c*e^14)*x)*sqrt((81*b^2*c^4*d^8 + 18*b^2*c^2*d^4*e^4 + b^2
*e^8 - 6*c^4*d^2*e^2*sqrt((81*b^4*c^4*d^8 + 18*b^4*c^2*d^4*e^4 + b^4*e^8)/c
^6))/(81*b^2*c^4*d^8 - 18*b^2*c^2*d^4*e^4 + b^2*e^8))*((81*b^4*c^4*d^8 + 18
*b^4*c^2*d^4*e^4 + b^4*e^8)/c^6)^(1/4) + (729*b^4*c^8*d^12 - 81*b^4*c^6*d^8
*e^4 - 9*b^4*c^4*d^4*e^8 + b^4*c^2*e^12)*sqrt((81*b^4*c^4*d^8 + 18*b^4*c^2*
d^4*e^4 + b^4*e^8)/c^6))*sqrt((81*b^2*c^4*d^8 + 18*b^2*c^2*d^4*e^4 + b^2*e^
8 - 6*c^4*d^2*e^2*sqrt((81*b^4*c^4*d^8 + 18*b^4*c^2*d^4*e^4 + b^4*e^8)/c^6)
)/(81*b^2*c^4*d^8 - 18*b^2*c^2*d^4*e^4 + b^2*e^8))*((81*b^4*c^4*d^8 + 18*b^
4*c^2*d^4*e^4 + b^4*e^8)/c^6)^(3/4) + sqrt(2)*((81*b^3*c^15*d^8*e^2 - b^3*c
^11*e^10)*x*sqrt((81*b^4*c^4*d^8 + 18*b^4*c^2*d^4*e^4 + b^4*e^8)/c^6)*sqrt(
(81*b^4*c^4*d^8 - 18*b^4*c^2*d^4*e^4 + b^4*e^8)/c^6) + 3*(729*b^5*c^15*d^14
 + 81*b^5*c^13*d^10*e^4 - 9*b^5*c^11*d^6*e^8 - b^5*c^9*d^2*e^12)*x*sqrt((81
*b^4*c^4*d^8 - 18*b^4*c^2*d^4*e^4 + b^4*e^8)/c^6))*sqrt((81*b^2*c^4*d^8 + 1
8*b^2*c^2*d^4*e^4 + b^2*e^8 - 6*c^4*d^2*e^2*sqrt((81*b^4*c^4*d^8 + 18*b^4*c
^2*d^4*e^4 + b^4*e^8)/c^6))/(81*b^2*c^4*d^8 - 18*b^2*c^2*d^4*e^4 + b^2*e^8)
)*((81*b^4*c^4*d^8 + 18*b^4*c^2*d^4*e^4 + b^4*e^8)/c^6)^(3/4) - (6561*b^6*c
^14*d^16 + 1458*b^6*c^12*d^12*e^4 - 18*b^6*c^8*d^4*e^12 - b^6*c^6*e^16)*sqr
t((81*b^4*c^4*d^8 + 18*b^4*c^2*d^4*e^4 + b^4*e^8)/c^6)*sqrt((81*b^4*c^4*d^8
 - 18*b^4*c^2*d^4*e^4 + b^4*e^8)/c^6))/(531441*b^10*c^12*d^24 + 118098*b^10
*c^10*d^20*e^4 - 6561*b^10*c^8*d^16*e^8 - 2916*b^10*c^6*d^12*e^12 - 81*b^10
*c^4*d^8*e^16 + 18*b^10*c^2*d^4*e^20 + b^10*e^24)) - 4*(81*a*b^4*c^5*d^8*e^
2 + 18*a*b^4*c^3*d^4*e^6 + a*b^4*c*e^10)*x^3 - 12*(81*a*b^4*c^5*d^9*e + 18*
a*b^4*c^3*d^5*e^5 + a*b^4*c*d*e^9)*x^2 - 4*(243*a*b^4*c^5*d^10 - 162*b^5*c^
4*d^8*e^2 + 54*a*b^4*c^3*d^6*e^4 - 36*b^5*c^2*d^4*e^6 + 3*a*b^4*c*d^2*e^8 -
 2*b^5*e^10)*x - 4*((81*b^5*c^5*d^8*e^2 + 18*b^5*c^3*d^4*e^6 + b^5*c*e^10)*
x^3 + 3*(81*b^5*c^5*d^9*e + 18*b^5*c^3*d^5*e^5 + b^5*c*d*e^9)*x^2 + 3*(81*b
^5*c^5*d^10 + 18*b^5*c^3*d^6*e^4 + b^5*c*d^2*e^8)*x)*arctan(c*x^2) + (486*b
^5*c^4*d^9*e + 108*b^5*c^2*d^5*e^5 + 6*b^5*d*e^9 - sqrt(2)*(81*b^4*c^5*d^8
```

```
+ 18*b^4*c^3*d^4*e^4 + b^4*c*e^8 + 6*b^2*c^5*d^2*e^2*sqrt((81*b^4*c^4*d^8 +
 18*b^4*c^2*d^4*e^4 + b^4*e^8)/c^6))*sqrt((81*b^2*c^4*d^8 + 18*b^2*c^2*d^4*
e^4 + b^2*e^8 - 6*c^4*d^2*e^2*sqrt((81*b^4*c^4*d^8 + 18*b^4*c^2*d^4*e^4 + b
^4*e^8)/c^6))/(81*b^2*c^4*d^8 - 18*b^2*c^2*d^4*e^4 + b^2*e^8))*((81*b^4*c^4
*d^8 + 18*b^4*c^2*d^4*e^4 + b^4*e^8)/c^6)^(1/4))*log((6561*b^6*c^8*d^16 - 1
62*b^6*c^4*d^8*e^8 + b^6*e^16)*x^2 + sqrt(2)*(3*(81*b^3*c^9*d^10 - 18*b^3*c
^7*d^6*e^4 + b^3*c^5*d^2*e^8)*x*sqrt((81*b^4*c^4*d^8 + 18*b^4*c^2*d^4*e^4 +
 b^4*e^8)/c^6) + (729*b^5*c^7*d^12*e^2 - 81*b^5*c^5*d^8*e^6 - 9*b^5*c^3*d^4
*e^10 + b^5*c*e^14)*x)*sqrt((81*b^2*c^4*d^8 + 18*b^2*c^2*d^4*e^4 + b^2*e^8
- 6*c^4*d^2*e^2*sqrt((81*b^4*c^4*d^8 + 18*b^4*c^2*d^4*e^4 + b^4*e^8)/c^6))/
(81*b^2*c^4*d^8 - 18*b^2*c^2*d^4*e^4 + b^2*e^8))*((81*b^4*c^4*d^8 + 18*b^4*
c^2*d^4*e^4 + b^4*e^8)/c^6)^(1/4) + (729*b^4*c^8*d^12 - 81*b^4*c^6*d^8*e^4
- 9*b^4*c^4*d^4*e^8 + b^4*c^2*e^12)*sqrt((81*b^4*c^4*d^8 + 18*b^4*c^2*d^4*e
^4 + b^4*e^8)/c^6)) + (486*b^5*c^4*d^9*e + 108*b^5*c^2*d^5*e^5 + 6*b^5*d*e^
9 + sqrt(2)*(81*b^4*c^5*d^8 + 18*b^4*c^3*d^4*e^4 + b^4*c*e^8 + 6*b^2*c^5*d^
2*e^2*sqrt((81*b^4*c^4*d^8 + 18*b^4*c^2*d^4*e^4 + b^4*e^8)/c^6))*sqrt((81*b
^2*c^4*d^8 + 18*b^2*c^2*d^4*e^4 + b^2*e^8 - 6*c^4*d^2*e^2*sqrt((81*b^4*c^4*
d^8 + 18*b^4*c^2*d^4*e^4 + b^4*e^8)/c^6))/(81*b^2*c^4*d^8 - 18*b^2*c^2*d^4*
e^4 + b^2*e^8))*((81*b^4*c^4*d^8 + 18*b^4*c^2*d^4*e^4 + b^4*e^8)/c^6)^(1/4)
)*log((6561*b^6*c^8*d^16 - 162*b^6*c^4*d^8*e^8 + b^6*e^16)*x^2 - sqrt(2)*(3
*(81*b^3*c^9*d^10 - 18*b^3*c^7*d^6*e^4 + b^3*c^5*d^2*e^8)*x*sqrt((81*b^4*c^
4*d^8 + 18*b^4*c^2*d^4*e^4 + b^4*e^8)/c^6) + (729*b^5*c^7*d^12*e^2 - 81*b^5
*c^5*d^8*e^6 - 9*b^5*c^3*d^4*e^10 + b^5*c*e^14)*x)*sqrt((81*b^2*c^4*d^8 + 1
8*b^2*c^2*d^4*e^4 + b^2*e^8 - 6*c^4*d^2*e^2*sqrt((81*b^4*c^4*d^8 + 18*b^4*c
^2*d^4*e^4 + b^4*e^8)/c^6))/(81*b^2*c^4*d^8 - 18*b^2*c^2*d^4*e^4 + b^2*e^8)
)*((81*b^4*c^4*d^8 + 18*b^4*c^2*d^4*e^4 + b^4*e^8)/c^6)^(1/4) + (729*b^4*c^
8*d^12 - 81*b^4*c^6*d^8*e^4 - 9*b^4*c^4*d^4*e^8 + b^4*c^2*e^12)*sqrt((81*b^
4*c^4*d^8 + 18*b^4*c^2*d^4*e^4 + b^4*e^8)/c^6)))/(81*b^4*c^5*d^8 + 18*b^4*c
^3*d^4*e^4 + b^4*c*e^8)
```

Sympy [A] time = 49.9778, size = 3135, normalized size = 12.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(a+b*atan(c*x**2)),x)
```

```
[Out] Piecewise((a*(d**2*x + d*e*x**2 + e**2*x**3/3), Eq(c, 0)), ((a - oo*I*b)*(d
**2*x + d*e*x**2 + e**2*x**3/3), Eq(c, -I/x**2)), ((a + oo*I*b)*(d**2*x + d
*e*x**2 + e**2*x**3/3), Eq(c, I/x**2)), (11*(-1)**(1/4)*a*c**10*e**2*x**4*(
c**(-2))**(17/4)*atan((-1)**(3/4)*x/(c**(-2))**(1/4))/(-24*I*c**6*x**4*(c**
```

```
(-2))**(3/2) - 24*I*c**4*(c**(-2))**(3/2)) + 11*(-1)**(1/4)*a*c**8*e**2*(c*
*(-2))**(17/4)*atan((-1)**(3/4)*x/(c**(-2))**(1/4))/(-24*I*c**6*x**4*(c**(-
2))**(3/2) - 24*I*c**4*(c**(-2))**(3/2)) - 11*(-1)**(3/4)*a*c**7*e**2*(c**(
-2))**(15/4)*atan(c*x**2)/(-24*I*c**6*x**4*(c**(-2))**(3/2) - 24*I*c**4*(c*
*(-2))**(3/2)) - 24*I*a*c**6*d**2*x**5*(c**(-2))**(3/2)/(-24*I*c**6*x**4*(c
**(-2))**(3/2) - 24*I*c**4*(c**(-2))**(3/2)) - 24*I*a*c**6*d*e*x**6*(c**(-2
))**(3/2)/(-24*I*c**6*x**4*(c**(-2))**(3/2) - 24*I*c**4*(c**(-2))**(3/2)) -
 8*I*a*c**6*e**2*x**7*(c**(-2))**(3/2)/(-24*I*c**6*x**4*(c**(-2))**(3/2) -
24*I*c**4*(c**(-2))**(3/2)) - 11*(-1)**(1/4)*a*c**6*e**2*x**4*(c**(-2))**(9
/4)*atan((-1)**(3/4)*x/(c**(-2))**(1/4))/(-24*I*c**6*x**4*(c**(-2))**(3/2)
- 24*I*c**4*(c**(-2))**(3/2)) - 24*I*a*c**4*d**2*x*(c**(-2))**(3/2)/(-24*I*
c**6*x**4*(c**(-2))**(3/2) - 24*I*c**4*(c**(-2))**(3/2)) - 24*I*a*c**4*d*e*
x**2*(c**(-2))**(3/2)/(-24*I*c**6*x**4*(c**(-2))**(3/2) - 24*I*c**4*(c**(-2
))**(3/2)) - 8*I*a*c**4*e**2*x**3*(c**(-2))**(3/2)/(-24*I*c**6*x**4*(c**(-2
))**(3/2) - 24*I*c**4*(c**(-2))**(3/2)) - 11*(-1)**(1/4)*a*c**4*e**2*(c**(-
2))**(9/4)*atan((-1)**(3/4)*x/(c**(-2))**(1/4))/(-24*I*c**6*x**4*(c**(-2))*
*(3/2) - 24*I*c**4*(c**(-2))**(3/2)) + 11*(-1)**(3/4)*a*c**3*e**2*(c**(-2))
**(7/4)*atan(c*x**2)/(-24*I*c**6*x**4*(c**(-2))**(3/2) - 24*I*c**4*(c**(-2)
)**(3/2)) + 66*(-1)**(1/4)*b*c**11*d**2*x**4*(c**(-2))**(17/4)*atan((-1)**(
3/4)*x/(c**(-2))**(1/4))/(-24*I*c**6*x**4*(c**(-2))**(3/2) - 24*I*c**4*(c**
(-2))**(3/2)) + 66*(-1)**(1/4)*b*c**9*d**2*(c**(-2))**(17/4)*atan((-1)**(3/
4)*x/(c**(-2))**(1/4))/(-24*I*c**6*x**4*(c**(-2))**(3/2) - 24*I*c**4*(c**(-
2))**(3/2)) - 114*(-1)**(3/4)*b*c**8*d**2*(c**(-2))**(15/4)*atan(c*x**2)/(-
24*I*c**6*x**4*(c**(-2))**(3/2) - 24*I*c**4*(c**(-2))**(3/2)) - 90*(-1)**(1
/4)*b*c**7*d**2*x**4*(c**(-2))**(9/4)*atan((-1)**(3/4)*x/(c**(-2))**(1/4))/
(-24*I*c**6*x**4*(c**(-2))**(3/2) - 24*I*c**4*(c**(-2))**(3/2)) - 24*I*b*c*
*6*d**2*x**5*(c**(-2))**(3/2)*atan(c*x**2)/(-24*I*c**6*x**4*(c**(-2))**(3/2
) - 24*I*c**4*(c**(-2))**(3/2)) - 24*I*b*c**6*d*e*x**6*(c**(-2))**(3/2)*ata
n(c*x**2)/(-24*I*c**6*x**4*(c**(-2))**(3/2) - 24*I*c**4*(c**(-2))**(3/2)) -
 8*I*b*c**6*e**2*x**7*(c**(-2))**(3/2)*atan(c*x**2)/(-24*I*c**6*x**4*(c**(-
2))**(3/2) - 24*I*c**4*(c**(-2))**(3/2)) + 24*(-1)**(1/4)*b*c**5*d**2*x**4*
(c**(-2))**(5/4)*log(x - (-1)**(1/4)*(c**(-2))**(1/4))/(-24*I*c**6*x**4*(c*
*(-2))**(3/2) - 24*I*c**4*(c**(-2))**(3/2)) - 12*(-1)**(1/4)*b*c**5*d**2*x*
*4*(c**(-2))**(5/4)*log(x**2 + I*sqrt(c**(-2)))/(-24*I*c**6*x**4*(c**(-2))*
*(3/2) - 24*I*c**4*(c**(-2))**(3/2)) - 90*(-1)**(1/4)*b*c**5*d**2*(c**(-2))
**(9/4)*atan((-1)**(3/4)*x/(c**(-2))**(1/4))/(-24*I*c**6*x**4*(c**(-2))**(3
/2) - 24*I*c**4*(c**(-2))**(3/2)) + 24*I*b*c**5*d*e*x**4*(c**(-2))**(3/2)*l
og(x**2 + I*sqrt(c**(-2)))/(-24*I*c**6*x**4*(c**(-2))**(3/2) - 24*I*c**4*(c
**(-2))**(3/2)) + 16*I*b*c**5*e**2*x**5*(c**(-2))**(3/2)/(-24*I*c**6*x**4*(
c**(-2))**(3/2) - 24*I*c**4*(c**(-2))**(3/2)) + 8*(-1)**(3/4)*b*c**5*e**2*x
**4*(c**(-2))**(7/4)*log(x - (-1)**(1/4)*(c**(-2))**(1/4))/(-24*I*c**6*x**4
*(c**(-2))**(3/2) - 24*I*c**4*(c**(-2))**(3/2)) - 4*(-1)**(3/4)*b*c**5*e**2
*x**4*(c**(-2))**(7/4)*log(x**2 + I*sqrt(c**(-2)))/(-24*I*c**6*x**4*(c**(-2
))**(3/2) - 24*I*c**4*(c**(-2))**(3/2)) + 8*(-1)**(3/4)*b*c**5*e**2*x**4*(c
**(-2))**(7/4)*atan((-1)**(3/4)*x/(c**(-2))**(1/4))/(-24*I*c**6*x**4*(c**(-
```

```
2))**(3/2) - 24*I*c**4*(c**(-2))**(3/2)) - 24*(-1)**(3/4)*b*c**4*d**2*x**4*
(c**(-2))**(3/4)*atan(c*x**2)/(-24*I*c**6*x**4*(c**(-2))**(3/2) - 24*I*c**4
*(c**(-2))**(3/2)) - 24*I*b*c**4*d**2*x*(c**(-2))**(3/2)*atan(c*x**2)/(-24*
I*c**6*x**4*(c**(-2))**(3/2) - 24*I*c**4*(c**(-2))**(3/2)) + 90*(-1)**(3/4)
*b*c**4*d**2*(c**(-2))**(7/4)*atan(c*x**2)/(-24*I*c**6*x**4*(c**(-2))**(3/2
) - 24*I*c**4*(c**(-2))**(3/2)) - 24*I*b*c**4*d*e*x**2*(c**(-2))**(3/2)*ata
n(c*x**2)/(-24*I*c**6*x**4*(c**(-2))**(3/2) - 24*I*c**4*(c**(-2))**(3/2)) -
 8*I*b*c**4*e**2*x**3*(c**(-2))**(3/2)*atan(c*x**2)/(-24*I*c**6*x**4*(c**(-
2))**(3/2) - 24*I*c**4*(c**(-2))**(3/2)) + 24*(-1)**(1/4)*b*c**3*d**2*(c**(
-2))**(5/4)*log(x - (-1)**(1/4)*(c**(-2))**(1/4))/(-24*I*c**6*x**4*(c**(-2)
)**(3/2) - 24*I*c**4*(c**(-2))**(3/2)) - 12*(-1)**(1/4)*b*c**3*d**2*(c**(-2
))**(5/4)*log(x**2 + I*sqrt(c**(-2)))/(-24*I*c**6*x**4*(c**(-2))**(3/2) - 2
4*I*c**4*(c**(-2))**(3/2)) + 24*I*b*c**3*d*e*(c**(-2))**(3/2)*log(x**2 + I*
sqrt(c**(-2)))/(-24*I*c**6*x**4*(c**(-2))**(3/2) - 24*I*c**4*(c**(-2))**(3/
2)) + 16*I*b*c**3*e**2*x*(c**(-2))**(3/2)/(-24*I*c**6*x**4*(c**(-2))**(3/2)
 - 24*I*c**4*(c**(-2))**(3/2)) + 8*(-1)**(3/4)*b*c**3*e**2*(c**(-2))**(7/4)
*log(x - (-1)**(1/4)*(c**(-2))**(1/4))/(-24*I*c**6*x**4*(c**(-2))**(3/2) -
24*I*c**4*(c**(-2))**(3/2)) - 4*(-1)**(3/4)*b*c**3*e**2*(c**(-2))**(7/4)*lo
g(x**2 + I*sqrt(c**(-2)))/(-24*I*c**6*x**4*(c**(-2))**(3/2) - 24*I*c**4*(c*
*(-2))**(3/2)) + 8*(-1)**(3/4)*b*c**3*e**2*(c**(-2))**(7/4)*atan((-1)**(3/4
)*x/(c**(-2))**(1/4))/(-24*I*c**6*x**4*(c**(-2))**(3/2) - 24*I*c**4*(c**(-2
))**(3/2)) - 24*b*c**2*d*e*x**4*atan(c*x**2)/(-24*I*c**6*x**4*(c**(-2))**(3
/2) - 24*I*c**4*(c**(-2))**(3/2)) + 8*(-1)**(1/4)*b*c**2*e**2*x**4*(c**(-2)
)**(1/4)*atan(c*x**2)/(-24*I*c**6*x**4*(c**(-2))**(3/2) - 24*I*c**4*(c**(-2
))**(3/2)) - 24*b*d*e*atan(c*x**2)/(-24*I*c**6*x**4*(c**(-2))**(3/2) - 24*I
*c**4*(c**(-2))**(3/2)) + 8*(-1)**(1/4)*b*e**2*(c**(-2))**(1/4)*atan(c*x**2
)/(-24*I*c**6*x**4*(c**(-2))**(3/2) - 24*I*c**4*(c**(-2))**(3/2)), True))
```

Giac [A] time = 1.8809, size = 510, normalized size = 2.04

$$\frac{1}{12}\,bc^5\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^6\sqrt{|c|}}+\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^6\sqrt{|c|}}+\frac{\sqrt{2}\log\left(x^2+\frac{\sqrt{2}x}{\sqrt{|c|}}+\frac{1}{|c|}\right)}{c^6\sqrt{|c|}}-\frac{\sqrt{2}\log\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arctan(c*x^2)),x, algorithm="giac")
```

```
[Out] 1/12*b*c^5*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(abs(c)))*sqrt(
abs(c)))/(c^6*sqrt(abs(c))) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)/s
qrt(abs(c)))*sqrt(abs(c)))/(c^6*sqrt(abs(c))) + sqrt(2)*log(x^2 + sqrt(2)*x
/sqrt(abs(c)) + 1/abs(c))/(c^6*sqrt(abs(c))) - sqrt(2)*log(x^2 - sqrt(2)*x/
```

```
sqrt(abs(c)) + 1/abs(c))/(c^6*sqrt(abs(c)))*e^2 - 1/4*b*c^3*d^2*(2*sqrt(2)
*sqrt(abs(c))*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(abs(c)))*sqrt(abs(c)))
/c^4 + 2*sqrt(2)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(2*x - sqrt(2)/sqrt(abs(c)
))*sqrt(abs(c)))/c^4 - sqrt(2)*sqrt(abs(c))*log(x^2 + sqrt(2)*x/sqrt(abs(c)
) + 1/abs(c))/c^4 + sqrt(2)*sqrt(abs(c))*log(x^2 - sqrt(2)*x/sqrt(abs(c)) +
 1/abs(c))/c^4) + 1/6*(2*b*c*x^3*arctan(c*x^2)*e^2 + 6*b*c*d*x^2*arctan(c*x
^2)*e + 6*b*c*d^2*x*arctan(c*x^2) + 2*a*c*x^3*e^2 + 6*a*c*d*x^2*e + 6*a*c*d
^2*x - 3*b*d*e*log(c^2*x^4 + 1) - 4*b*x*e^2)/c
```

3.22 $\int (d+ex)\left(a+b\tan^{-1}\left(cx^2\right)\right)\,dx$

Optimal. Leaf size=192

$$\frac{(d+ex)^2\left(a+b\tan^{-1}\left(cx^2\right)\right)}{2e}-\frac{be\log\left(c^2x^4+1\right)}{4c}-\frac{bd^2\tan^{-1}\left(cx^2\right)}{2e}-\frac{bd\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}\sqrt{c}}+\frac{bd\log\left(cx^2+\sqrt{2}\sqrt{c}x\right.}{2\sqrt{2}\sqrt{c}}$$

```
[Out] -(b*d^2*ArcTan[c*x^2])/(2*e) + ((d + e*x)^2*(a + b*ArcTan[c*x^2]))/(2*e) +
(b*d*ArcTan[1 - Sqrt[2]*Sqrt[c]*x])/(Sqrt[2]*Sqrt[c]) - (b*d*ArcTan[1 + Sqr
t[2]*Sqrt[c]*x])/(Sqrt[2]*Sqrt[c]) - (b*d*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2
])/(2*Sqrt[2]*Sqrt[c]) + (b*d*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(2*Sqrt[2
]*Sqrt[c]) - (b*e*Log[1 + c^2*x^4])/(4*c)
```

Rubi [A] time = 0.208994, antiderivative size = 191, normalized size of antiderivative = 0.99, number of steps used = 16, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6742, 5027, 297, 1162, 617, 204, 1165, 628, 5033, 260}

$$\frac{a(d+ex)^2}{2e}-\frac{be\log\left(c^2x^4+1\right)}{4c}-\frac{bd\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}\sqrt{c}}+\frac{bd\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}\sqrt{c}}+bdx\tan^{-1}\left(cx^2\right)+\frac{bd\tan^{-1}\left(\right.}{\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] (a*(d + e*x)^2)/(2*e) + b*d*x*ArcTan[c*x^2] + (b*e*x^2*ArcTan[c*x^2])/2 + (
b*d*ArcTan[1 - Sqrt[2]*Sqrt[c]*x])/(Sqrt[2]*Sqrt[c]) - (b*d*ArcTan[1 + Sqrt
[2]*Sqrt[c]*x])/(Sqrt[2]*Sqrt[c]) - (b*d*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2]
)/(2*Sqrt[2]*Sqrt[c]) + (b*d*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(2*Sqrt[2]
*Sqrt[c]) - (b*e*Log[1 + c^2*x^4])/(4*c)
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 5027

```
Int[ArcTan[(c_.)*(x_)^(n_)], x_Symbol] :> Simp[x*ArcTan[c*x^n], x] - Dist[c
*n, Int[x^n/(1 + c^2*x^(2*n)), x], x] /; FreeQ[{c, n}, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
```

```
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int (d+ex)\left(a+b\tan^{-1}\left(cx^2\right)\right)dx &= \int \left(a(d+ex)+b(d+ex)\tan^{-1}\left(cx^2\right)\right)dx \\
&= \frac{a(d+ex)^2}{2e} + b\int (d+ex)\tan^{-1}\left(cx^2\right)dx \\
&= \frac{a(d+ex)^2}{2e} + b\int \left(d\tan^{-1}\left(cx^2\right)+ex\tan^{-1}\left(cx^2\right)\right)dx \\
&= \frac{a(d+ex)^2}{2e} + (bd)\int \tan^{-1}\left(cx^2\right)dx + (be)\int x\tan^{-1}\left(cx^2\right)dx \\
&= \frac{a(d+ex)^2}{2e} + bdx\tan^{-1}\left(cx^2\right) + \frac{1}{2}bex^2\tan^{-1}\left(cx^2\right) - (2bcd)\int \frac{x^2}{1+c^2x^4}dx - (bce)\int \\
&= \frac{a(d+ex)^2}{2e} + bdx\tan^{-1}\left(cx^2\right) + \frac{1}{2}bex^2\tan^{-1}\left(cx^2\right) - \frac{be\log\left(1+c^2x^4\right)}{4c} + (bd)\int \frac{1}{1+} \\
&= \frac{a(d+ex)^2}{2e} + bdx\tan^{-1}\left(cx^2\right) + \frac{1}{2}bex^2\tan^{-1}\left(cx^2\right) - \frac{be\log\left(1+c^2x^4\right)}{4c} - \frac{(bd)\int \frac{}{\frac{1}{c}-}}{2c} \\
&= \frac{a(d+ex)^2}{2e} + bdx\tan^{-1}\left(cx^2\right) + \frac{1}{2}bex^2\tan^{-1}\left(cx^2\right) - \frac{bd\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{2\sqrt{2}\sqrt{c}} + \frac{b}{} \\
&= \frac{a(d+ex)^2}{2e} + bdx\tan^{-1}\left(cx^2\right) + \frac{1}{2}bex^2\tan^{-1}\left(cx^2\right) + \frac{bd\tan^{-1}\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}} - \frac{bd\,\text{ta}}{}
\end{aligned}$$

Mathematica [A] time = 0.0953756, size = 153, normalized size = 0.8

$$adx + \frac{1}{2}aex^2 - \frac{be\log\left(c^2x^4+1\right)}{4c} + bdx\tan^{-1}\left(cx^2\right) - \frac{bd\left(\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)-\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)-2\tan^{-1}\left(1-\right.\right.}{2\sqrt{2}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + b*d*x*ArcTan[c*x^2] + (b*e*x^2*ArcTan[c*x^2])/2 - (b*
d*(-2*ArcTan[1 - Sqrt[2]*Sqrt[c]*x] + 2*ArcTan[1 + Sqrt[2]*Sqrt[c]*x] + Log
[1 - Sqrt[2]*Sqrt[c]*x + c*x^2] - Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2]))/(2*S
qrt[2]*Sqrt[c]) - (b*e*Log[1 + c^2*x^4])/(4*c)
```

Maple [A] time = 0.028, size = 167, normalized size = 0.9

$$\frac{ax^2e}{2} + adx + \frac{b\arctan\left(cx^2\right)x^2e}{2} + b\arctan\left(cx^2\right)dx - \frac{bd\sqrt{2}}{4c}\ln\left(\left(x^2 - \sqrt[4]{c^{-2}}x\sqrt{2} + \sqrt{c^{-2}}\right)\left(x^2 + \sqrt[4]{c^{-2}}x\sqrt{2} + \sqrt{c^{-2}}\right)^{-1}\right)\sqrt[4]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(a+b*arctan(c*x^2)),x)
```

```
[Out] 1/2*a*x^2*e+a*d*x+1/2*b*arctan(c*x^2)*x^2*e+b*arctan(c*x^2)*d*x-1/4*b*d/c/(
1/c^2)^(1/4)*2^(1/2)*ln((x^2-(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2))/(x^2+(1
/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2)))-1/2*b*d/c/(1/c^2)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(1/c^2)^(1/4)*x+1)-1/2*b*d/c/(1/c^2)^(1/4)*2^(1/2)*arctan(2^(1/2)
/(1/c^2)^(1/4)*x-1)-1/4*b*e*ln(c^2*x^4+1)/c
```

Maxima [A] time = 1.50104, size = 404, normalized size = 2.1

$$\frac{1}{2}aex^2 + \frac{1}{4}\left(c\left(\frac{\sqrt{2}\log\left(\sqrt{c^2}x^2 + \sqrt{2}\left(c^2\right)^{\frac{1}{4}}x + 1\right)}{\left(c^2\right)^{\frac{3}{4}}} - \frac{\sqrt{2}\log\left(\sqrt{c^2}x^2 - \sqrt{2}\left(c^2\right)^{\frac{1}{4}}x + 1\right)}{\left(c^2\right)^{\frac{3}{4}}} - \frac{\sqrt{2}\log\left(\frac{2\sqrt{c^2}x - \sqrt{2}\sqrt{-\sqrt{c^2}} + \sqrt{2}\left(c^2\right)^{\frac{1}{4}}}{2\sqrt{c^2}x + \sqrt{2}\sqrt{-\sqrt{c^2}} + \sqrt{2}\left(c^2\right)^{\frac{1}{4}}}\right)}{\sqrt{c^2}\sqrt{-\sqrt{c^2}}} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/2*a*e*x^2 + 1/4*(c*(sqrt(2)*log(sqrt(c^2)*x^2 + sqrt(2)*(c^2)^(1/4)*x + 1
)/(c^2)^(3/4) - sqrt(2)*log(sqrt(c^2)*x^2 - sqrt(2)*(c^2)^(1/4)*x + 1)/(c^2
)^(3/4) - sqrt(2)*log((2*sqrt(c^2)*x - sqrt(2)*sqrt(-sqrt(c^2)) + sqrt(2)*(
c^2)^(1/4))/(2*sqrt(c^2)*x + sqrt(2)*sqrt(-sqrt(c^2)) + sqrt(2)*(c^2)^(1/4)
))/(sqrt(c^2)*sqrt(-sqrt(c^2))) - sqrt(2)*log((2*sqrt(c^2)*x - sqrt(2)*sqrt
(-sqrt(c^2)) - sqrt(2)*(c^2)^(1/4))/(2*sqrt(c^2)*x + sqrt(2)*sqrt(-sqrt(c^2
```

```
)) - sqrt(2)*(c^2)^(1/4)))/(sqrt(c^2)*sqrt(-sqrt(c^2)))) + 4*x*arctan(c*x^2
))*b*d + a*d*x + 1/4*(2*c*x^2*arctan(c*x^2) - log(c^2*x^4 + 1))*b*e/c
```

Fricas [B] time = 2.81062, size = 1166, normalized size = 6.07

$$2\,ab^4cd^4ex^2 + 4\,ab^4cd^5x + 4\,\sqrt{2}\left(\frac{b^4d^4}{c^2}\right)^{\frac{1}{4}} b^4cd^4 \arctan\left(-\frac{b^8d^8+\sqrt{2}\left(\frac{b^4d^4}{c^2}\right)^{\frac{5}{4}} b^3c^3d^3x-\sqrt{2}\sqrt{b^6d^6x^2+\sqrt{2}\left(\frac{b^4d^4}{c^2}\right)^{\frac{3}{4}} b^3cd^3x+\sqrt{\frac{b^4d^4}{c^2}}b^4d^4}\left(\frac{b^4d^4}{c^2}\right)^{\frac{5}{4}} c^3}{b^8d^8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctan(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*b^4*c*d^4*e*x^2 + 4*a*b^4*c*d^5*x + 4*sqrt(2)*(b^4*d^4/c^2)^(1/4)*
b^4*c*d^4*arctan(-(b^8*d^8 + sqrt(2)*(b^4*d^4/c^2)^(5/4)*b^3*c^3*d^3*x - sq
rt(2)*sqrt(b^6*d^6*x^2 + sqrt(2)*(b^4*d^4/c^2)^(3/4)*b^3*c*d^3*x + sqrt(b^4
*d^4/c^2)*b^4*d^4)*(b^4*d^4/c^2)^(5/4)*c^3)/(b^8*d^8)) + 4*sqrt(2)*(b^4*d^4
/c^2)^(1/4)*b^4*c*d^4*arctan((b^8*d^8 - sqrt(2)*(b^4*d^4/c^2)^(5/4)*b^3*c^3
*d^3*x + sqrt(2)*sqrt(b^6*d^6*x^2 - sqrt(2)*(b^4*d^4/c^2)^(3/4)*b^3*c*d^3*x
 + sqrt(b^4*d^4/c^2)*b^4*d^4)*(b^4*d^4/c^2)^(5/4)*c^3)/(b^8*d^8)) + 2*(b^5*
c*d^4*e*x^2 + 2*b^5*c*d^5*x)*arctan(c*x^2) - (b^5*d^4*e - sqrt(2)*(b^4*d^4/
c^2)^(1/4)*b^4*c*d^4)*log(b^6*d^6*x^2 + sqrt(2)*(b^4*d^4/c^2)^(3/4)*b^3*c*d
^3*x + sqrt(b^4*d^4/c^2)*b^4*d^4) - (b^5*d^4*e + sqrt(2)*(b^4*d^4/c^2)^(1/4
)*b^4*c*d^4)*log(b^6*d^6*x^2 - sqrt(2)*(b^4*d^4/c^2)^(3/4)*b^3*c*d^3*x + sq
rt(b^4*d^4/c^2)*b^4*d^4))/(b^4*c*d^4)
```

Sympy [A] time = 31.7067, size = 1515, normalized size = 7.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*atan(c*x**2)),x)
```

```
[Out] Piecewise((a*(d*x + e*x**2/2), Eq(c, 0)), ((a - oo*I*b)*(d*x + e*x**2/2), E
q(c, -I/x**2)), ((a + oo*I*b)*(d*x + e*x**2/2), Eq(c, I/x**2)), (-2*I*a*c**
11*d*x**5*(c**(-2))**(11/2)/(-2*I*c**19*x**4*(c**(-2))**(19/2) - 2*I*c**17*
```

```
(c**(-2))**(19/2)) - I*a*c**11*e*x**6*(c**(-2))**(11/2)/(-2*I*c**19*x**4*(c
**(-2))**(19/2) - 2*I*c**17*(c**(-2))**(19/2)) - 2*I*a*c**9*d*x*(c**(-2))**
(11/2)/(-2*I*c**19*x**4*(c**(-2))**(19/2) - 2*I*c**17*(c**(-2))**(19/2)) -
I*a*c**9*e*x**2*(c**(-2))**(11/2)/(-2*I*c**19*x**4*(c**(-2))**(19/2) - 2*I*
c**17*(c**(-2))**(19/2)) - 2*(-1)**(1/4)*b*c**18*d*x**4*(c**(-2))**(37/4)*a
tan((-1)**(3/4)*x/(c**(-2))**(1/4))/(-2*I*c**19*x**4*(c**(-2))**(19/2) - 2*
I*c**17*(c**(-2))**(19/2)) - 2*(-1)**(1/4)*b*c**16*d*(c**(-2))**(37/4)*atan
((-1)**(3/4)*x/(c**(-2))**(1/4))/(-2*I*c**19*x**4*(c**(-2))**(19/2) - 2*I*c
**17*(c**(-2))**(19/2)) + 2*(-1)**(1/4)*b*c**14*d*x**4*(c**(-2))**(29/4)*lo
g(x - (-1)**(1/4)*(c**(-2))**(1/4))/(-2*I*c**19*x**4*(c**(-2))**(19/2) - 2*
I*c**17*(c**(-2))**(19/2)) + 2*(-1)**(1/4)*b*c**12*d*(c**(-2))**(29/4)*log(
x - (-1)**(1/4)*(c**(-2))**(1/4))/(-2*I*c**19*x**4*(c**(-2))**(19/2) - 2*I*
c**17*(c**(-2))**(19/2)) - 2*I*b*c**11*d*x**5*(c**(-2))**(11/2)*atan(c*x**2
)/(-2*I*c**19*x**4*(c**(-2))**(19/2) - 2*I*c**17*(c**(-2))**(19/2)) - I*b*c
**11*e*x**6*(c**(-2))**(11/2)*atan(c*x**2)/(-2*I*c**19*x**4*(c**(-2))**(19/
2) - 2*I*c**17*(c**(-2))**(19/2)) - 2*I*b*c**9*d*x*(c**(-2))**(11/2)*atan(c
*x**2)/(-2*I*c**19*x**4*(c**(-2))**(19/2) - 2*I*c**17*(c**(-2))**(19/2)) -
I*b*c**9*e*x**2*(c**(-2))**(11/2)*atan(c*x**2)/(-2*I*c**19*x**4*(c**(-2))**
(19/2) - 2*I*c**17*(c**(-2))**(19/2)) - 2*(-1)**(3/4)*b*c**5*d*x**4*(c**(-2
))**(11/4)*atan(c*x**2)/(-2*I*c**19*x**4*(c**(-2))**(19/2) - 2*I*c**17*(c**
(-2))**(19/2)) - 2*(-1)**(1/4)*b*c**4*d*x**4*(c**(-2))**(9/4)*log(x**2 + I*
sqrt(c**(-2)))/(-2*I*c**19*x**4*(c**(-2))**(19/2) - 2*I*c**17*(c**(-2))**(1
9/2)) - 2*(-1)**(3/4)*b*c**3*d*(c**(-2))**(11/4)*atan(c*x**2)/(-2*I*c**19*x
**4*(c**(-2))**(19/2) - 2*I*c**17*(c**(-2))**(19/2)) + (-1)**(1/4)*b*c**2*d
*x**4*(c**(-2))**(5/4)*log(x**2 + I*sqrt(c**(-2)))/(-2*I*c**19*x**4*(c**(-2
))**(19/2) - 2*I*c**17*(c**(-2))**(19/2)) - 2*(-1)**(1/4)*b*c**2*d*(c**(-2)
)**(9/4)*log(x**2 + I*sqrt(c**(-2)))/(-2*I*c**19*x**4*(c**(-2))**(19/2) - 2
*I*c**17*(c**(-2))**(19/2)) + I*b*c**2*e*x**4*(c**(-2))**(3/2)*log(x**2 + I
*sqrt(c**(-2)))/(-2*I*c**19*x**4*(c**(-2))**(19/2) - 2*I*c**17*(c**(-2))**(
19/2)) + (-1)**(1/4)*b*d*(c**(-2))**(5/4)*log(x**2 + I*sqrt(c**(-2)))/(-2*I
*c**19*x**4*(c**(-2))**(19/2) - 2*I*c**17*(c**(-2))**(19/2)) - b*e*x**4*ata
n(c*x**2)/(-2*I*c**20*x**4*(c**(-2))**(19/2) - 2*I*c**18*(c**(-2))**(19/2))
 - b*e*atan(c*x**2)/(-2*I*c**22*x**4*(c**(-2))**(19/2) - 2*I*c**20*(c**(-2)
)**(19/2)) + I*b*e*(c**(-2))**(3/2)*log(x**2 + I*sqrt(c**(-2)))/(-2*I*c**19
*x**4*(c**(-2))**(19/2) - 2*I*c**17*(c**(-2))**(19/2)), True))
```

Giac [A] time = 1.26275, size = 271, normalized size = 1.41

$$-\frac{1}{4}\,bc^3d\left(\frac{2\,\sqrt{2}\sqrt{|c|}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^4}+\frac{2\,\sqrt{2}\sqrt{|c|}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^4}-\frac{\sqrt{2}\sqrt{|c|}\log\left(x^2+\frac{\sqrt{2}x}{\sqrt{|c|}}+\frac{1}{|c|}\right)}{c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctan(c*x^2)),x, algorithm="giac")
```

```
[Out] -1/4*b*c^3*d*(2*sqrt(2)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt
(abs(c)))*sqrt(abs(c)))/c^4 + 2*sqrt(2)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(2*
x - sqrt(2)/sqrt(abs(c)))*sqrt(abs(c)))/c^4 - sqrt(2)*sqrt(abs(c))*log(x^2
+ sqrt(2)*x/sqrt(abs(c)) + 1/abs(c))/c^4 + sqrt(2)*sqrt(abs(c))*log(x^2 - s
qrt(2)*x/sqrt(abs(c)) + 1/abs(c))/c^4) + 1/4*(2*b*c*x^2*arctan(c*x^2)*e + 4
*b*c*d*x*arctan(c*x^2) + 2*a*c*x^2*e + 4*a*c*d*x - b*e*log(c^2*x^4 + 1))/c
```

3.23 $\int \frac{a+b\tan^{-1}\left(cx^2\right)}{d+ex}\,dx$

Optimal. Leaf size=501

$$\frac{bc\text{PolyLog}\left(2,\frac{\sqrt[4]{-c^2}(d+ex)}{\sqrt[4]{-c^2}d-e}\right)}{2\sqrt{-c^2}e}-\frac{bc\text{PolyLog}\left(2,\frac{\sqrt{-\sqrt{-c^2}}(d+ex)}{\sqrt{-\sqrt{-c^2}}d-e}\right)}{2\sqrt{-c^2}e}+\frac{bc\text{PolyLog}\left(2,\frac{\sqrt[4]{-c^2}(d+ex)}{\sqrt[4]{-c^2}d+e}\right)}{2\sqrt{-c^2}e}-\frac{bc\text{PolyLog}\left(2,\frac{\sqrt{-\sqrt{-c^2}}(d+ex)}{\sqrt{-\sqrt{-c^2}}d+e}\right)}{2\sqrt{-c^2}e}+$$

```
[Out] ((a + b*ArcTan[c*x^2])*Log[d + e*x])/e + (b*c*Log[(e*(1 - (-c^2)^(1/4)*x))/
((-c^2)^(1/4)*d + e)]*Log[d + e*x])/(2*Sqrt[-c^2]*e) + (b*c*Log[-((e*(1 + (
-c^2)^(1/4)*x))/((-c^2)^(1/4)*d - e))]*Log[d + e*x])/(2*Sqrt[-c^2]*e) - (b*
c*Log[(e*(1 - Sqrt[-Sqrt[-c^2]]*x))/(Sqrt[-Sqrt[-c^2]]*d + e)]*Log[d + e*x]
)/(2*Sqrt[-c^2]*e) - (b*c*Log[-((e*(1 + Sqrt[-Sqrt[-c^2]]*x))/(Sqrt[-Sqrt[-
c^2]]*d - e))]*Log[d + e*x])/(2*Sqrt[-c^2]*e) + (b*c*PolyLog[2, ((-c^2)^(1/
4)*(d + e*x))/((-c^2)^(1/4)*d - e)])/(2*Sqrt[-c^2]*e) - (b*c*PolyLog[2, (Sq
rt[-Sqrt[-c^2]]*(d + e*x))/(Sqrt[-Sqrt[-c^2]]*d - e)])/(2*Sqrt[-c^2]*e) + (
b*c*PolyLog[2, ((-c^2)^(1/4)*(d + e*x))/((-c^2)^(1/4)*d + e)])/(2*Sqrt[-c^2
]*e) - (b*c*PolyLog[2, (Sqrt[-Sqrt[-c^2]]*(d + e*x))/(Sqrt[-Sqrt[-c^2]]*d +
 e)])/(2*Sqrt[-c^2]*e)
```

Rubi [F] time = 0.0637799, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{a+b\tan^{-1}\left(cx^2\right)}{d+ex}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTan[c*x^2])/(d + e*x),x]
```

```
[Out] (a*Log[d + e*x])/e + b*Defer[Int][ArcTan[c*x^2]/(d + e*x), x]
```

Rubi steps

$$\int \frac{a + b\tan^{-1}\left(cx^2\right)}{d + ex}\,dx = \int \left(\frac{a}{d + ex} + \frac{b\tan^{-1}\left(cx^2\right)}{d + ex}\right) dx$$
$$= \frac{a\log(d + ex)}{e} + b\int \frac{\tan^{-1}\left(cx^2\right)}{d + ex}\,dx$$

Mathematica [C] time = 32.123, size = 326, normalized size = 0.65

$$\frac{a\log(d + ex)}{e} + \frac{b\left(2\tan^{-1}\left(cx^2\right)\log(d + ex) + i\left(\text{PolyLog}\left(2, \frac{\sqrt{c}(d+ex)}{\sqrt{c}d - \sqrt[4]{-1}e}\right) + \text{PolyLog}\left(2, \frac{\sqrt{c}(d+ex)}{\sqrt{c}d + \sqrt[4]{-1}e}\right) - \text{PolyLog}\left(2, \frac{\sqrt{c}(d\right.\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^2])/(d + e*x),x]
```

```
[Out] (a*Log[d + e*x])/e + (b*(2*ArcTan[c*x^2]*Log[d + e*x] + I*(Log[d + e*x]*Log
[1 - (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - (-1)^(1/4)*e)] + Log[d + e*x]*Log[1 -
 (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + (-1)^(1/4)*e)] - Log[d + e*x]*Log[1 - (Sq
rt[c]*(d + e*x))/(Sqrt[c]*d - (-1)^(3/4)*e)] - Log[d + e*x]*Log[1 - (Sqrt[c
]*(d + e*x))/(Sqrt[c]*d + (-1)^(3/4)*e)] + PolyLog[2, (Sqrt[c]*(d + e*x))/(
Sqrt[c]*d - (-1)^(1/4)*e)] + PolyLog[2, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + (-
1)^(1/4)*e)] - PolyLog[2, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - (-1)^(3/4)*e)] -
 PolyLog[2, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + (-1)^(3/4)*e)])))/(2*e)
```

Maple [C] time = 0.125, size = 138, normalized size = 0.3

$$\frac{a\ln(ex + d)}{e} + \frac{b\ln(ex + d)\arctan\left(cx^2\right)}{e} - \frac{be}{2c}\sum_{\_R1=\text{RootOf}\left(c^2\_Z^4 - 4c^2d\_Z^3 + 6c^2d^2\_Z^2 - 4c^2d^3\_Z + c^2d^4 + e^4\right)} \frac{1}{-\_R1^2 - 2\,\_R1\,d + d^2}\left($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))/(e*x+d),x)
```

```
[Out] a*ln(e*x+d)/e+b*ln(e*x+d)/e*arctan(c*x^2)-1/2*b*e/c*sum(1/(_R1^2-2*_R1*d+d^
2)*(ln(e*x+d)*ln((-e*x+_R1-d)/_R1)+dilog((-e*x+_R1-d)/_R1)),_R1=RootOf(_Z^4
*c^2-4*_Z^3*c^2*d+6*_Z^2*c^2*d^2-4*_Z*c^2*d^3+c^2*d^4+e^4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$2\,b\int \frac{\arctan\left(cx^2\right)}{2\,(ex+d)}\,dx+\frac{a\log\left(ex+d\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/(e*x+d),x, algorithm="maxima")
```

```
[Out] 2*b*integrate(1/2*arctan(c*x^2)/(e*x + d), x) + a*log(e*x + d)/e
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\arctan\left(cx^2\right)+a}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x^2) + a)/(e*x + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\arctan\left(cx^2\right)+a}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)/(e*x + d), x)
```

3.24 $\int \frac{a+b\tan^{-1}\left(cx^2\right)}{(d+ex)^2}\,dx$

Optimal. Leaf size=328

$$-\frac{a+b\tan^{-1}\left(cx^2\right)}{e(d+ex)}+\frac{bcde\log\left(c^2x^4+1\right)}{2\left(c^2d^4+e^4\right)}-\frac{b\sqrt{c}\left(cd^2+e^2\right)\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}\left(c^2d^4+e^4\right)}+\frac{b\sqrt{c}\left(cd^2+e^2\right)\log\left(cx^2+\sqrt{2}\sqrt{c}x+\right.}{2\sqrt{2}\left(c^2d^4+e^4\right)}$$

```
[Out] (b*c^2*d^3*ArcTan[c*x^2])/(e*(c^2*d^4 + e^4)) - (a + b*ArcTan[c*x^2])/(e*(d
 + e*x)) + (b*Sqrt[c]*(c*d^2 - e^2)*ArcTan[1 - Sqrt[2]*Sqrt[c]*x])/(Sqrt[2]
*(c^2*d^4 + e^4)) - (b*Sqrt[c]*(c*d^2 - e^2)*ArcTan[1 + Sqrt[2]*Sqrt[c]*x])
/(Sqrt[2]*(c^2*d^4 + e^4)) - (2*b*c*d*e*Log[d + e*x])/(c^2*d^4 + e^4) - (b*
Sqrt[c]*(c*d^2 + e^2)*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2])/(2*Sqrt[2]*(c^2*d
^4 + e^4)) + (b*Sqrt[c]*(c*d^2 + e^2)*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(
2*Sqrt[2]*(c^2*d^4 + e^4)) + (b*c*d*e*Log[1 + c^2*x^4])/(2*(c^2*d^4 + e^4))
```

Rubi [A] time = 0.522159, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 14, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {5205, 12, 6725, 1876, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 203, 260}

$$-\frac{a+b\tan^{-1}\left(cx^2\right)}{e(d+ex)}+\frac{bcde\log\left(c^2x^4+1\right)}{2\left(c^2d^4+e^4\right)}-\frac{b\sqrt{c}\left(cd^2+e^2\right)\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}\left(c^2d^4+e^4\right)}+\frac{b\sqrt{c}\left(cd^2+e^2\right)\log\left(cx^2+\sqrt{2}\sqrt{c}x+\right.}{2\sqrt{2}\left(c^2d^4+e^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])/(d + e*x)^2,x]
```

```
[Out] (b*c^2*d^3*ArcTan[c*x^2])/(e*(c^2*d^4 + e^4)) - (a + b*ArcTan[c*x^2])/(e*(d
 + e*x)) + (b*Sqrt[c]*(c*d^2 - e^2)*ArcTan[1 - Sqrt[2]*Sqrt[c]*x])/(Sqrt[2]
*(c^2*d^4 + e^4)) - (b*Sqrt[c]*(c*d^2 - e^2)*ArcTan[1 + Sqrt[2]*Sqrt[c]*x])
/(Sqrt[2]*(c^2*d^4 + e^4)) - (2*b*c*d*e*Log[d + e*x])/(c^2*d^4 + e^4) - (b*
Sqrt[c]*(c*d^2 + e^2)*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2])/(2*Sqrt[2]*(c^2*d
^4 + e^4)) + (b*Sqrt[c]*(c*d^2 + e^2)*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(
2*Sqrt[2]*(c^2*d^4 + e^4)) + (b*c*d*e*Log[1 + c^2*x^4])/(2*(c^2*d^4 + e^4))
```

Rule 5205

```
Int[((a_.) + ArcTan[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((c + d*x)^(m + 1)*(a + b*ArcTan[u]))/(d*(m + 1)), x] - Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 + u^2), x], x], x]
```

```
/; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&
  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m +
1, x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}\left(cx^2\right)}{(d+ex)^2}\,dx = -\frac{a+b\tan^{-1}\left(cx^2\right)}{e(d+ex)} + \frac{b\int \frac{2cx}{(d+ex)\left(1+c^2x^4\right)}\,dx}{e}$$

$$= -\frac{a+b\tan^{-1}\left(cx^2\right)}{e(d+ex)} + \frac{(2bc)\int \frac{x}{(d+ex)\left(1+c^2x^4\right)}\,dx}{e}$$

$$= -\frac{a+b\tan^{-1}\left(cx^2\right)}{e(d+ex)} + \frac{(2bc)\int \left(-\frac{de^3}{\left(c^2d^4+e^4\right)(d+ex)} + \frac{e^3+c^2d^3x-c^2d^2ex^2+c^2de^2x^3}{\left(c^2d^4+e^4\right)\left(1+c^2x^4\right)}\right)dx}{e}$$

$$= -\frac{a+b\tan^{-1}\left(cx^2\right)}{e(d+ex)} - \frac{2bcde\log(d+ex)}{c^2d^4+e^4} + \frac{(2bc)\int \frac{e^3+c^2d^3x-c^2d^2ex^2+c^2de^2x^3}{1+c^2x^4}\,dx}{e\left(c^2d^4+e^4\right)}$$

$$= -\frac{a+b\tan^{-1}\left(cx^2\right)}{e(d+ex)} - \frac{2bcde\log(d+ex)}{c^2d^4+e^4} + \frac{(2bc)\int \left(\frac{e^3-c^2d^2ex^2}{1+c^2x^4} + \frac{x\left(c^2d^3+c^2de^2x^2\right)}{1+c^2x^4}\right)dx}{e\left(c^2d^4+e^4\right)}$$

$$= -\frac{a+b\tan^{-1}\left(cx^2\right)}{e(d+ex)} - \frac{2bcde\log(d+ex)}{c^2d^4+e^4} + \frac{(2bc)\int \frac{e^3-c^2d^2ex^2}{1+c^2x^4}\,dx}{e\left(c^2d^4+e^4\right)} + \frac{(2bc)\int \frac{x\left(c^2d^3+c^2de^2x^2\right)}{1+c^2x^4}\,dx}{e\left(c^2d^4+e^4\right)}$$

$$= -\frac{a+b\tan^{-1}\left(cx^2\right)}{e(d+ex)} - \frac{2bcde\log(d+ex)}{c^2d^4+e^4} + \frac{(bc)\,\mathrm{Subst}\left(\int \frac{c^2d^3+c^2de^2x}{1+c^2x^2}\,dx, x, x^2\right)}{e\left(c^2d^4+e^4\right)} - \frac{\left(b\left(cd^2-e^2\right)\right)}{c^2d^4}$$

$$= -\frac{a+b\tan^{-1}\left(cx^2\right)}{e(d+ex)} - \frac{2bcde\log(d+ex)}{c^2d^4+e^4} + \frac{\left(bc^3d^3\right)\mathrm{Subst}\left(\int \frac{1}{1+c^2x^2}\,dx, x, x^2\right)}{e\left(c^2d^4+e^4\right)} + \frac{\left(bc^3de\right)\mathrm{Subs}}{c^2}$$

$$= \frac{bc^2d^3\tan^{-1}\left(cx^2\right)}{e\left(c^2d^4+e^4\right)} - \frac{a+b\tan^{-1}\left(cx^2\right)}{e(d+ex)} - \frac{2bcde\log(d+ex)}{c^2d^4+e^4} - \frac{b\sqrt{c}\left(cd^2+e^2\right)\log\left(1-\sqrt{2}\sqrt{c}x\right.}{2\sqrt{2}\left(c^2d^4+e^4\right)}$$

$$= \frac{bc^2d^3\tan^{-1}\left(cx^2\right)}{e\left(c^2d^4+e^4\right)} - \frac{a+b\tan^{-1}\left(cx^2\right)}{e(d+ex)} + \frac{b\sqrt{c}\left(cd^2-e^2\right)\tan^{-1}\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\left(c^2d^4+e^4\right)} - \frac{b\sqrt{c}\left(cd^2-e^2\right)}{\sqrt{2}\left(}$$

Mathematica [A] time = 0.744945, size = 321, normalized size = 0.98

$$-\frac{4a\left(c^2d^4+e^4\right)+4b\left(c^2d^4+e^4\right)\tan^{-1}\left(cx^2\right)+2b\sqrt{c}\left(2c^{3/2}d^3-\sqrt{2}cd^2e+\sqrt{2}e^3\right)\tan^{-1}\left(1-\sqrt{2}\sqrt{c}x\right)(d+ex)+2b\sqrt{c}\left(2c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^2])/(d + e*x)^2,x]
```

```
[Out] -(4*a*(c^2*d^4 + e^4) + 4*b*(c^2*d^4 + e^4)*ArcTan[c*x^2] + 2*b*Sqrt[c]*(2*
c^(3/2)*d^3 - Sqrt[2]*c*d^2*e + Sqrt[2]*e^3)*(d + e*x)*ArcTan[1 - Sqrt[2]*S
qrt[c]*x] + 2*b*Sqrt[c]*(2*c^(3/2)*d^3 + Sqrt[2]*c*d^2*e - Sqrt[2]*e^3)*(d
+ e*x)*ArcTan[1 + Sqrt[2]*Sqrt[c]*x] + 8*b*c*d*e^2*(d + e*x)*Log[d + e*x] +
 Sqrt[2]*b*Sqrt[c]*e*(c*d^2 + e^2)*(d + e*x)*Log[1 - Sqrt[2]*Sqrt[c]*x + c*
x^2] - Sqrt[2]*b*Sqrt[c]*e*(c*d^2 + e^2)*(d + e*x)*Log[1 + Sqrt[2]*Sqrt[c]*
x + c*x^2] - 2*b*c*d*e^2*(d + e*x)*Log[1 + c^2*x^4])/(4*e*(c^2*d^4 + e^4)*(
d + e*x))
```

Maple [A] time = 0.036, size = 433, normalized size = 1.3

$$-\frac{a}{(ex+d)\,e}-\frac{b\arctan\left(cx^2\right)}{(ex+d)\,e}-2\,\frac{bcde\ln(ex+d)}{c^2d^4+e^4}+\frac{be^2c\sqrt{2}}{2\,c^2d^4+2\,e^4}\sqrt[4]{c^{-2}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{c^{-2}}}+1\right)+\frac{be^2c\sqrt{2}}{2\,c^2d^4+2\,e^4}\sqrt[4]{c^{-2}}\arcta$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))/(e*x+d)^2,x)
```

```
[Out] -a/(e*x+d)/e-b/(e*x+d)/e*arctan(c*x^2)-2*b*c*d*e*ln(e*x+d)/(c^2*d^4+e^4)+1/
2*b*e^2*c/(c^2*d^4+e^4)*(1/c^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c^2)^(1/4)*
x+1)+1/2*b*e^2*c/(c^2*d^4+e^4)*(1/c^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c^2)
^(1/4)*x-1)+1/4*b*e^2*c/(c^2*d^4+e^4)*(1/c^2)^(1/4)*2^(1/2)*ln((x^2+(1/c^2)
^(1/4)*x*2^(1/2)+(1/c^2)^(1/2))/(x^2-(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2))
)+b/e*c^3/(c^2*d^4+e^4)*d^3/(c^2)^(1/2)*arctan(x^2*(c^2)^(1/2))-1/4*b*c/(c^
2*d^4+e^4)*d^2/(1/c^2)^(1/4)*2^(1/2)*ln((x^2-(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2
)^(1/2))/(x^2+(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2)))-1/2*b*c/(c^2*d^4+e^4)
*d^2/(1/c^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c^2)^(1/4)*x+1)-1/2*b*c/(c^2*d
^4+e^4)*d^2/(1/c^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c^2)^(1/4)*x-1)+1/2*b*c
*d*e*ln(c^2*x^4+1)/(c^2*d^4+e^4)
```

Maxima [A] time = 1.48466, size = 649, normalized size = 1.98

$$-\frac{1}{4}\left(\left(\left(\frac{8\,de\log(ex+d)}{c^2d^4+e^4}-\frac{\sqrt{2}\left(\sqrt{2}\left(c^2\right)^{\frac{1}{4}}c^2de^2+\sqrt{c^2}c^2d^2e+c^2e^3\right)\log\left(\sqrt{c^2}x^2+\sqrt{2}\left(c^2\right)^{\frac{1}{4}}x+1\right)}{\left(c^2\right)^{\frac{1}{4}}c^2}+\frac{\sqrt{2}\left(\sqrt{2}\left(c^2\right)^{\frac{1}{4}}c^2de^2-\sqrt{c^2}c^2d^2e-c^2e^3\right)\log\left(\sqrt{c^2}x^2-\sqrt{2}\left(c^2\right)^{\frac{1}{4}}x+\right.}{\left(c^2\right)^{\frac{1}{4}}c^2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/4*((8*d*e*log(e*x + d)/(c^2*d^4 + e^4) - (sqrt(2)*(sqrt(2)*(c^2)^(1/4)*c
^2*d*e^2 + sqrt(c^2)*c^2*d^2*e + c^2*e^3)*log(sqrt(c^2)*x^2 + sqrt(2)*(c^2)
^(1/4)*x + 1)/((c^2)^(1/4)*c^2) + sqrt(2)*(sqrt(2)*(c^2)^(1/4)*c^2*d*e^2 -
sqrt(c^2)*c^2*d^2*e - c^2*e^3)*log(sqrt(c^2)*x^2 - sqrt(2)*(c^2)^(1/4)*x +
1)/((c^2)^(1/4)*c^2) - (2*c^4*d^3 + sqrt(2)*(c^2)^(3/4)*c^2*d^2*e - sqrt(2)
*(c^2)^(1/4)*c^2*e^3)*log((2*sqrt(c^2)*x - sqrt(2)*sqrt(-sqrt(c^2)) + sqrt(
2)*(c^2)^(1/4))/(2*sqrt(c^2)*x + sqrt(2)*sqrt(-sqrt(c^2)) + sqrt(2)*(c^2)^(
1/4)))/((c^2)^(1/4)*c^2*sqrt(-sqrt(c^2))) + (2*c^4*d^3 - sqrt(2)*(c^2)^(3/4
)*c^2*d^2*e + sqrt(2)*(c^2)^(1/4)*c^2*e^3)*log((2*sqrt(c^2)*x - sqrt(2)*sqr
t(-sqrt(c^2)) - sqrt(2)*(c^2)^(1/4))/(2*sqrt(c^2)*x + sqrt(2)*sqrt(-sqrt(c^
2)) - sqrt(2)*(c^2)^(1/4)))/((c^2)^(1/4)*c^2*sqrt(-sqrt(c^2))))/(c^2*d^4*e
+ e^5))*c + 4*arctan(c*x^2)/(e^2*x + d*e))*b - a/(e^2*x + d*e)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 14.2967, size = 512, normalized size = 1.56

$$\frac{1}{2}\left(\left(\frac{de\log\left(c^2x^4+1\right)}{c^2d^4+e^4}-\frac{4de^2\log\left(|xe+d|\right)}{c^2d^4e+e^5}-\frac{2\left(\sqrt{2}c^2d^3|c|+c^2d^2\sqrt{|c|}e-|c|^{\frac{3}{2}}e^3\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{\sqrt{2}c^4d^4e+\sqrt{2}c^2e^5}+\frac{2\left(\sqrt{2}c^2d\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/2*((d*e*log(c^2*x^4 + 1)/(c^2*d^4 + e^4) - 4*d*e^2*log(abs(x*e + d))/(c^2
*d^4*e + e^5) - 2*(sqrt(2)*c^2*d^3*abs(c) + c^2*d^2*sqrt(abs(c))*e - abs(c)
^(3/2)*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(abs(c)))*sqrt(abs(c)))/(
sqrt(2)*c^4*d^4*e + sqrt(2)*c^2*e^5) + 2*(sqrt(2)*c^2*d^3*abs(c) - c^2*d^2*
sqrt(abs(c))*e + abs(c)^(3/2)*e^3)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)/sqrt(a
bs(c)))*sqrt(abs(c)))/(sqrt(2)*c^4*d^4*e + sqrt(2)*c^2*e^5) + (c^2*d^2*sqrt
(abs(c)) + abs(c)^(3/2)*e^2)*log(x^2 + sqrt(2)*x/sqrt(abs(c)) + 1/abs(c))/(
sqrt(2)*c^4*d^4 + sqrt(2)*c^2*e^4) - (c^2*d^2*sqrt(abs(c)) + abs(c)^(3/2)*e
^2)*log(x^2 - sqrt(2)*x/sqrt(abs(c)) + 1/abs(c))/(sqrt(2)*c^4*d^4 + sqrt(2)
*c^2*e^4))*c - 2*arctan(c*x^2)*e^(-1)/(x*e + d))*b - a*e^(-1)/(x*e + d)
```

3.25 $\int (d+ex)\left(a+b\tan^{-1}\left(cx^2\right)\right)^2\,dx$

Optimal. Leaf size=1325

result too large to display

```
[Out] a^2*d*x - (2*(-1)^(3/4)*a*b*d*ArcTan[(-1)^(3/4)*Sqrt[c]*x])/Sqrt[c] + ((-1)
^(3/4)*b^2*d*ArcTan[(-1)^(3/4)*Sqrt[c]*x]^2)/Sqrt[c] + ((I/2)*e*(a + b*ArcT
an[c*x^2])^2)/c + (e*x^2*(a + b*ArcTan[c*x^2])^2)/2 + (2*(-1)^(3/4)*a*b*d*A
rcTanh[(-1)^(3/4)*Sqrt[c]*x])/Sqrt[c] - ((-1)^(1/4)*b^2*d*ArcTanh[(-1)^(3/4
)*Sqrt[c]*x]^2)/Sqrt[c] + (2*(-1)^(1/4)*b^2*d*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*
Log[2/(1 - (-1)^(1/4)*Sqrt[c]*x)])/Sqrt[c] - (2*(-1)^(1/4)*b^2*d*ArcTan[(-1
)^(3/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/Sqrt[c] + ((-1)^(1/4)
*b^2*d*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[(Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/
(1 + (-1)^(1/4)*Sqrt[c]*x)])/Sqrt[c] + (2*(-1)^(1/4)*b^2*d*ArcTanh[(-1)^(3/
4)*Sqrt[c]*x]*Log[2/(1 - (-1)^(3/4)*Sqrt[c]*x)])/Sqrt[c] - (2*(-1)^(1/4)*b^
2*d*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(3/4)*Sqrt[c]*x)])/Sqrt[c
] + ((-1)^(1/4)*b^2*d*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[-((Sqrt[2]*((-1)^(3
/4) + Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x))])/Sqrt[c] + ((-1)^(1/4)*b^2*d
*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1
+ (-1)^(3/4)*Sqrt[c]*x)])/Sqrt[c] + ((-1)^(1/4)*b^2*d*ArcTan[(-1)^(3/4)*Sqr
t[c]*x]*Log[((1 - I)*(1 + (-1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)
])/Sqrt[c] + I*a*b*d*x*Log[1 - I*c*x^2] + ((-1)^(1/4)*b^2*d*ArcTan[(-1)^(3/
4)*Sqrt[c]*x]*Log[1 - I*c*x^2])/Sqrt[c] - ((-1)^(1/4)*b^2*d*ArcTanh[(-1)^(3
/4)*Sqrt[c]*x]*Log[1 - I*c*x^2])/Sqrt[c] - (b^2*d*x*Log[1 - I*c*x^2]^2)/4 +
 (b*e*(a + b*ArcTan[c*x^2])*Log[2/(1 + I*c*x^2)])/c - I*a*b*d*x*Log[1 + I*c
*x^2] - ((-1)^(1/4)*b^2*d*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/Sq
rt[c] + ((-1)^(1/4)*b^2*d*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/S
qrt[c] + (b^2*d*x*Log[1 - I*c*x^2]*Log[1 + I*c*x^2])/2 - (b^2*d*x*Log[1 + I
*c*x^2]^2)/4 + ((-1)^(3/4)*b^2*d*PolyLog[2, 1 - 2/(1 - (-1)^(1/4)*Sqrt[c]*x
)])/Sqrt[c] + ((-1)^(3/4)*b^2*d*PolyLog[2, 1 - 2/(1 + (-1)^(1/4)*Sqrt[c]*x)
])/Sqrt[c] - ((-1)^(3/4)*b^2*d*PolyLog[2, 1 - (Sqrt[2]*((-1)^(1/4) + Sqrt[c
]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/(2*Sqrt[c]) + ((-1)^(1/4)*b^2*d*PolyLog[
2, 1 - 2/(1 - (-1)^(3/4)*Sqrt[c]*x)])/Sqrt[c] + ((-1)^(1/4)*b^2*d*PolyLog[2
, 1 - 2/(1 + (-1)^(3/4)*Sqrt[c]*x)])/Sqrt[c] - ((-1)^(1/4)*b^2*d*PolyLog[2,
 1 + (Sqrt[2]*((-1)^(3/4) + Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)])/(2*Sqr
t[c]) - ((-1)^(1/4)*b^2*d*PolyLog[2, 1 - ((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x
))/(1 + (-1)^(3/4)*Sqrt[c]*x)])/(2*Sqrt[c]) - ((-1)^(3/4)*b^2*d*PolyLog[2,
1 - ((1 - I)*(1 + (-1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/(2*Sq
rt[c]) + ((I/2)*b^2*e*PolyLog[2, 1 - 2/(1 + I*c*x^2)])/c
```

Rubi [A] time = 3.09881, antiderivative size = 1554, normalized size of antiderivative =

1.17, number of steps used = 110, number of rules used = 46, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.556, Rules used = {6742, 5027, 297, 1162, 617, 204, 1165, 628, 5033, 260, 5029, 2450, 2476, 2448, 321, 203, 2470, 12, 4920, 4854, 2402, 2315, 206, 2556, 205, 4928, 4856, 2447, 208, 5992, 5920, 5984, 5918, 5035, 2454, 2389, 2296, 2295, 30, 2557, 2475, 43, 2416, 2394, 2393, 2391}

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Int[(d + e*x)*(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] (a^2*(d + e*x)^2)/(2*e) + ((-1)^(3/4)*b^2*d*ArcTan[(-1)^(3/4)*Sqrt[c]*x]^2)
/Sqrt[c] + 2*a*b*d*x*ArcTan[c*x^2] + a*b*e*x^2*ArcTan[c*x^2] + (Sqrt[2]*a*b
*d*ArcTan[1 - Sqrt[2]*Sqrt[c]*x])/Sqrt[c] - (Sqrt[2]*a*b*d*ArcTan[1 + Sqrt[
2]*Sqrt[c]*x])/Sqrt[c] - ((-1)^(1/4)*b^2*d*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]^2)
/Sqrt[c] + (2*(-1)^(1/4)*b^2*d*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 - (-1)
^(1/4)*Sqrt[c]*x)])/Sqrt[c] - (2*(-1)^(1/4)*b^2*d*ArcTan[(-1)^(3/4)*Sqrt[c]
*x]*Log[2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/Sqrt[c] + ((-1)^(1/4)*b^2*d*ArcTan[(
-1)^(3/4)*Sqrt[c]*x]*Log[(Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1 + (-1)^(1/4)
*Sqrt[c]*x)])/Sqrt[c] + (2*(-1)^(1/4)*b^2*d*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*L
og[2/(1 - (-1)^(3/4)*Sqrt[c]*x)])/Sqrt[c] - (2*(-1)^(1/4)*b^2*d*ArcTanh[(-1
)^(3/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(3/4)*Sqrt[c]*x)])/Sqrt[c] + ((-1)^(1/4)
*b^2*d*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[-((Sqrt[2]*((-1)^(3/4) + Sqrt[c]*x
))/(1 + (-1)^(3/4)*Sqrt[c]*x))])/Sqrt[c] + ((-1)^(1/4)*b^2*d*ArcTanh[(-1)^(
3/4)*Sqrt[c]*x]*Log[((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(3/4)*Sq
rt[c]*x)])/Sqrt[c] + ((-1)^(1/4)*b^2*d*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[((1
 - I)*(1 + (-1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/Sqrt[c] + ((
-1)^(1/4)*b^2*d*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[1 - I*c*x^2])/Sqrt[c] - ((
-1)^(1/4)*b^2*d*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 - I*c*x^2])/Sqrt[c] - (
b^2*d*x*Log[1 - I*c*x^2]^2)/4 - ((I/8)*b^2*e*(1 - I*c*x^2)*Log[1 - I*c*x^2]
^2)/c - ((I/4)*b^2*e*Log[1 - I*c*x^2]*Log[(1 + I*c*x^2)/2])/c - ((-1)^(1/4)
*b^2*d*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/Sqrt[c] + ((-1)^(1/4)
*b^2*d*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/Sqrt[c] + ((I/4)*b^2
*e*Log[(1 - I*c*x^2)/2]*Log[1 + I*c*x^2])/c + (b^2*d*x*Log[1 - I*c*x^2]*Log
[1 + I*c*x^2])/2 + (b^2*e*x^2*Log[1 - I*c*x^2]*Log[1 + I*c*x^2])/4 - (b^2*d
*x*Log[1 + I*c*x^2]^2)/4 + ((I/8)*b^2*e*(1 + I*c*x^2)*Log[1 + I*c*x^2]^2)/c
 - (a*b*d*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2])/(Sqrt[2]*Sqrt[c]) + (a*b*d*Lo
g[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(Sqrt[2]*Sqrt[c]) - (a*b*e*Log[1 + c^2*x^
4])/(2*c) - ((I/4)*b^2*e*PolyLog[2, (1 - I*c*x^2)/2])/c + ((I/4)*b^2*e*Poly
Log[2, (1 + I*c*x^2)/2])/c + ((-1)^(3/4)*b^2*d*PolyLog[2, 1 - 2/(1 - (-1)^(
1/4)*Sqrt[c]*x)])/Sqrt[c] + ((-1)^(3/4)*b^2*d*PolyLog[2, 1 - 2/(1 + (-1)^(1
/4)*Sqrt[c]*x)])/Sqrt[c] - ((-1)^(3/4)*b^2*d*PolyLog[2, 1 - (Sqrt[2]*((-1)^
(1/4) + Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/(2*Sqrt[c]) + ((-1)^(1/4)*
b^2*d*PolyLog[2, 1 - 2/(1 - (-1)^(3/4)*Sqrt[c]*x)])/Sqrt[c] + ((-1)^(1/4)*b
```

```
^2*d*PolyLog[2, 1 - 2/(1 + (-1)^(3/4)*Sqrt[c]*x)])/Sqrt[c] - ((-1)^(1/4)*b^
2*d*PolyLog[2, 1 + (Sqrt[2]*((-1)^(3/4) + Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[
c]*x)])/(2*Sqrt[c]) - ((-1)^(1/4)*b^2*d*PolyLog[2, 1 - ((1 + I)*(1 + (-1)^(
1/4)*Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)])/(2*Sqrt[c]) - ((-1)^(3/4)*b^2
*d*PolyLog[2, 1 - ((1 - I)*(1 + (-1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt
[c]*x)])/(2*Sqrt[c])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 5027

```
Int[ArcTan[(c_.)*(x_)^(n_)], x_Symbol] :> Simp[x*ArcTan[c*x^n], x] - Dist[c
*n, Int[x^n/(1 + c^2*x^(2*n)), x], x] /; FreeQ[{c, n}, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5029

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Int[Expand
Integrand[(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*Log[1 + I*c*x^n])/2)^p, x],
x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] && IntegerQ[n]
```

Rule 2450

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[(x^n*
(a + b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
```

```
& IntegerQ[s]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
```

```
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2556

```
Int[Log[v_]*Log[w_], x_Symbol] :> Simp[x*Log[v]*Log[w], x] + (-Int[Simplify
Integrand[(x*Log[w]*D[v, x])/v, x], x] - Int[SimplifyIntegrand[(x*Log[v]*D[
w, x])/w, x], x]) /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w,
x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
```

```
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5035

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*L
og[1 + I*c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
 && IntegerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int (d+ex)\left(a+b\tan^{-1}\left(cx^2\right)\right)^2\,dx = \int \left(a^2(d+ex)+2ab(d+ex)\tan^{-1}\left(cx^2\right)+b^2(d+ex)\tan^{-1}\left(cx^2\right)^2\right)\,dx$$

$$= \frac{a^2(d+ex)^2}{2e}+(2ab)\int (d+ex)\tan^{-1}\left(cx^2\right)\,dx+b^2\int (d+ex)\tan^{-1}\left(cx^2\right)^2\,dx$$

$$= \frac{a^2(d+ex)^2}{2e}+(2ab)\int \left(d\tan^{-1}\left(cx^2\right)+ex\tan^{-1}\left(cx^2\right)\right)\,dx+b^2\int \left(d\tan^{-1}\left(cx^2\right)^2+\right.$$

$$= \frac{a^2(d+ex)^2}{2e}+(2abd)\int \tan^{-1}\left(cx^2\right)\,dx+\left(b^2d\right)\int \tan^{-1}\left(cx^2\right)^2\,dx+(2abe)\int x\tan$$

$$= \frac{a^2(d+ex)^2}{2e}+2abdx\tan^{-1}\left(cx^2\right)+abex^2\tan^{-1}\left(cx^2\right)+\left(b^2d\right)\int \left(-\frac{1}{4}\log^2\left(1-icx^2\right)\right.$$

$$= \frac{a^2(d+ex)^2}{2e}+2abdx\tan^{-1}\left(cx^2\right)+abex^2\tan^{-1}\left(cx^2\right)-\frac{abe\log\left(1+c^2x^4\right)}{2c}+(2abd)$$

$$= \frac{a^2(d+ex)^2}{2e}+2abdx\tan^{-1}\left(cx^2\right)+abex^2\tan^{-1}\left(cx^2\right)-\frac{1}{4}b^2dx\log^2\left(1-icx^2\right)+\frac{1}{2}b^2$$

$$= \frac{a^2(d+ex)^2}{2e}+2abdx\tan^{-1}\left(cx^2\right)+abex^2\tan^{-1}\left(cx^2\right)-\frac{1}{4}b^2dx\log^2\left(1-icx^2\right)+\frac{1}{2}b^2$$

$$= \frac{a^2(d+ex)^2}{2e}+2abdx\tan^{-1}\left(cx^2\right)+abex^2\tan^{-1}\left(cx^2\right)+\frac{\sqrt{2}abd\tan^{-1}\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{c}}$$

$$= -\frac{1}{2}b^2ex^2+\frac{a^2(d+ex)^2}{2e}+2abdx\tan^{-1}\left(cx^2\right)+abex^2\tan^{-1}\left(cx^2\right)+\frac{\sqrt{2}abd\tan^{-1}\left(1\right.}{\sqrt{c}}$$

$$= -4b^2dx-\frac{1}{2}b^2ex^2+\frac{a^2(d+ex)^2}{2e}+2abdx\tan^{-1}\left(cx^2\right)+abex^2\tan^{-1}\left(cx^2\right)+\frac{\sqrt{2}abd}{}$$

$$= -\frac{1}{2}b^2ex^2+\frac{a^2(d+ex)^2}{2e}-\frac{2\sqrt[4]{-1}b^2d\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}}+\frac{(-1)^{3/4}b^2d\tan^{-1}\left((-1)^{3/4}\right.}{\sqrt{c}}$$

$$= \frac{a^2(d+ex)^2}{2e}+\frac{(-1)^{3/4}b^2d\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)^2}{\sqrt{c}}+2abdx\tan^{-1}\left(cx^2\right)+abex^2\tan^{-1}\left(c\right.$$

$$= \frac{a^2(d+ex)^2}{2e}+\frac{(-1)^{3/4}b^2d\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)^2}{\sqrt{c}}+2abdx\tan^{-1}\left(cx^2\right)+abex^2\tan^{-1}\left(c\right.$$

$$= \frac{a^2(d+ex)^2}{2e}+\frac{(-1)^{3/4}b^2d\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)^2}{\sqrt{c}}+2abdx\tan^{-1}\left(cx^2\right)+abex^2\tan^{-1}\left(c\right.$$

$$= \frac{a^2(d+ex)^2}{2e}+\frac{(-1)^{3/4}b^2d\tan^{-1}\left((-1)^{3/4}\sqrt{c}x\right)^2}{\sqrt{c}}+2abdx\tan^{-1}\left(cx^2\right)+abex^2\tan^{-1}\left(c\right.$$

Mathematica [C] time = 32.1986, size = 5593, normalized size = 4.22

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)*(a + b*ArcTan[c*x^2])^2,x]

[Out] Result too large to show
```

Maple [F] time = 0.277, size = 0, normalized size = 0.

$$\int (ex+d)\left(a+b\arctan\left(cx^2\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(a+b*arctan(c*x^2))^2,x)

[Out] int((e*x+d)*(a+b*arctan(c*x^2))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctan(c*x^2))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(a^2ex+a^2d+\left(b^2ex+b^2d\right)\arctan\left(cx^2\right)^2+2\left(abex+abd\right)\arctan\left(cx^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctan(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*e*x + a^2*d + (b^2*e*x + b^2*d)*arctan(c*x^2)^2 + 2*(a*b*e*x +
 a*b*d)*arctan(c*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b \operatorname{atan}\left(cx^2\right)\right)^2 (d + ex)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*atan(c*x**2))**2,x)
```

```
[Out] Integral((a + b*atan(c*x**2))**2*(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex + d)\left(b \arctan\left(cx^2\right) + a\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctan(c*x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(b*arctan(c*x^2) + a)^2, x)
```

3.26 $\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{d+ex}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{d+ex},x\right)$$

```
[Out] Unintegrable[(a + b*ArcTan[c*x^2])^2/(d + e*x), x]
```

Rubi [A] time = 0.123024, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{d+ex}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTan[c*x^2])^2/(d + e*x),x]
```

```
[Out] (a^2*Log[d + e*x])/e + 2*a*b*Defer[Int][ArcTan[c*x^2]/(d + e*x), x] + b^2*D
efer[Int][ArcTan[c*x^2]^2/(d + e*x), x]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{d+ex}\,dx &= \int\left(\frac{a^2}{d+ex}+\frac{2ab\tan^{-1}\left(cx^2\right)}{d+ex}+\frac{b^2\tan^{-1}\left(cx^2\right)^2}{d+ex}\right)dx\\ &= \frac{a^2\log(d+ex)}{e}+(2ab)\int\frac{\tan^{-1}\left(cx^2\right)}{d+ex}\,dx+b^2\int\frac{\tan^{-1}\left(cx^2\right)^2}{d+ex}\,dx\end{aligned}$$

Mathematica [A] time = 70.5899, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{d+ex}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x^2])^2/(d + e*x),x]

[Out] Integrate[(a + b*ArcTan[c*x^2])^2/(d + e*x), x]
```

Maple [A] time = 0.543, size = 0, normalized size = 0.

$$\int \frac{\left(a + b \arctan\left(cx^2\right)\right)^2}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))^2/(e*x+d),x)

[Out] int((a+b*arctan(c*x^2))^2/(e*x+d),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{a^2 \log\left(ex + d\right)}{e} + \int \frac{b^2 \arctan\left(cx^2\right)^2 + 2\, ab \arctan\left(cx^2\right)}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/(e*x+d),x, algorithm="maxima")

[Out] a^2*log(e*x + d)/e + integrate((b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2))/
(e*x + d), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \arctan\left(cx^2\right)^2 + 2\, ab \arctan\left(cx^2\right) + a^2}{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2)/(e*x + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))**2/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \arctan\left(cx^2\right) + a\right)^2}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^2/(e*x + d), x)
```

3.27 $\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{(d+ex)^2}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{(d+ex)^2},x\right)$$

```
[Out] Unintegrable[(a + b*ArcTan[c*x^2])^2/(d + e*x)^2, x]
```

Rubi [A] time = 0.65645, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{(d+ex)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTan[c*x^2])^2/(d + e*x)^2,x]
```

```
[Out] -(a^2/(e*(d + e*x))) + (2*a*b*c^2*d^3*ArcTan[c*x^2])/(e*(c^2*d^4 + e^4)) -
(2*a*b*ArcTan[c*x^2])/(e*(d + e*x)) + (Sqrt[2]*a*b*Sqrt[c]*(c*d^2 - e^2)*Ar
cTan[1 - Sqrt[2]*Sqrt[c]*x])/(c^2*d^4 + e^4) - (Sqrt[2]*a*b*Sqrt[c]*(c*d^2
- e^2)*ArcTan[1 + Sqrt[2]*Sqrt[c]*x])/(c^2*d^4 + e^4) - (4*a*b*c*d*e*Log[d
+ e*x])/(c^2*d^4 + e^4) - (a*b*Sqrt[c]*(c*d^2 + e^2)*Log[1 - Sqrt[2]*Sqrt[c
]*x + c*x^2])/(Sqrt[2]*(c^2*d^4 + e^4)) + (a*b*Sqrt[c]*(c*d^2 + e^2)*Log[1
+ Sqrt[2]*Sqrt[c]*x + c*x^2])/(Sqrt[2]*(c^2*d^4 + e^4)) + (a*b*c*d*e*Log[1
+ c^2*x^4])/(c^2*d^4 + e^4) + b^2*Defer[Int][ArcTan[c*x^2]^2/(d + e*x)^2, x
]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{(d+ex)^2}\,dx = \int\left(\frac{a^2}{(d+ex)^2}+\frac{2ab\tan^{-1}\left(cx^2\right)}{(d+ex)^2}+\frac{b^2\tan^{-1}\left(cx^2\right)^2}{(d+ex)^2}\right)dx$$

$$= -\frac{a^2}{e(d+ex)}+(2ab)\int\frac{\tan^{-1}\left(cx^2\right)}{(d+ex)^2}\,dx+b^2\int\frac{\tan^{-1}\left(cx^2\right)^2}{(d+ex)^2}\,dx$$

$$= -\frac{a^2}{e(d+ex)}-\frac{2ab\tan^{-1}\left(cx^2\right)}{e(d+ex)}+b^2\int\frac{\tan^{-1}\left(cx^2\right)^2}{(d+ex)^2}\,dx+\frac{(2ab)\int\frac{2cx}{(d+ex)\left(1+c^2x^4\right)}\,dx}{e}$$

$$= -\frac{a^2}{e(d+ex)}-\frac{2ab\tan^{-1}\left(cx^2\right)}{e(d+ex)}+b^2\int\frac{\tan^{-1}\left(cx^2\right)^2}{(d+ex)^2}\,dx+\frac{(4abc)\int\frac{x}{(d+ex)\left(1+c^2x^4\right)}\,dx}{e}$$

$$= -\frac{a^2}{e(d+ex)}-\frac{2ab\tan^{-1}\left(cx^2\right)}{e(d+ex)}+b^2\int\frac{\tan^{-1}\left(cx^2\right)^2}{(d+ex)^2}\,dx+\frac{(4abc)\int\left(-\frac{de^3}{\left(c^2d^4+e^4\right)(d+ex)}+\frac{e^3+c^2d}{\left(c^2\right.}\right.}{e}$$

$$= -\frac{a^2}{e(d+ex)}-\frac{2ab\tan^{-1}\left(cx^2\right)}{e(d+ex)}-\frac{4abcde\log(d+ex)}{c^2d^4+e^4}+b^2\int\frac{\tan^{-1}\left(cx^2\right)^2}{(d+ex)^2}\,dx+\frac{(4abc)\int\frac{e^3}{}}{e}$$

$$= -\frac{a^2}{e(d+ex)}-\frac{2ab\tan^{-1}\left(cx^2\right)}{e(d+ex)}-\frac{4abcde\log(d+ex)}{c^2d^4+e^4}+b^2\int\frac{\tan^{-1}\left(cx^2\right)^2}{(d+ex)^2}\,dx+\frac{(4abc)\int\left(\frac{e^3}{}\right.}{}$$

$$= -\frac{a^2}{e(d+ex)}-\frac{2ab\tan^{-1}\left(cx^2\right)}{e(d+ex)}-\frac{4abcde\log(d+ex)}{c^2d^4+e^4}+b^2\int\frac{\tan^{-1}\left(cx^2\right)^2}{(d+ex)^2}\,dx+\frac{(4abc)\int\frac{e^3}{}}{e\left(c^2d^4\right.}$$

$$= -\frac{a^2}{e(d+ex)}-\frac{2ab\tan^{-1}\left(cx^2\right)}{e(d+ex)}-\frac{4abcde\log(d+ex)}{c^2d^4+e^4}+b^2\int\frac{\tan^{-1}\left(cx^2\right)^2}{(d+ex)^2}\,dx+\frac{(2abc)\,\mathrm{Sub}}{}$$

$$= -\frac{a^2}{e(d+ex)}-\frac{2ab\tan^{-1}\left(cx^2\right)}{e(d+ex)}-\frac{4abcde\log(d+ex)}{c^2d^4+e^4}+b^2\int\frac{\tan^{-1}\left(cx^2\right)^2}{(d+ex)^2}\,dx+\frac{\left(2abc^3d^3\right)\mathrm{S}}{}$$

$$= -\frac{a^2}{e(d+ex)}+\frac{2abc^2d^3\tan^{-1}\left(cx^2\right)}{e\left(c^2d^4+e^4\right)}-\frac{2ab\tan^{-1}\left(cx^2\right)}{e(d+ex)}-\frac{4abcde\log(d+ex)}{c^2d^4+e^4}-\frac{ab\sqrt{c}\left(cd^2+\right.}{}$$

$$= -\frac{a^2}{e(d+ex)}+\frac{2abc^2d^3\tan^{-1}\left(cx^2\right)}{e\left(c^2d^4+e^4\right)}-\frac{2ab\tan^{-1}\left(cx^2\right)}{e(d+ex)}+\frac{\sqrt{2}ab\sqrt{c}\left(cd^2-e^2\right)\tan^{-1}\left(1-\sqrt{2}\right.}{c^2d^4+e^4}$$

Mathematica [A] time = 63.7161, size = 0, normalized size = 0.

$$\int \frac{\left(a + b \tan^{-1}\left(cx^2\right)\right)^2}{(d + ex)^2}\, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x^2])^2/(d + e*x)^2,x]

[Out] Integrate[(a + b*ArcTan[c*x^2])^2/(d + e*x)^2, x]
```

Maple [A] time = 0.49, size = 0, normalized size = 0.

$$\int \frac{\left(a + b \arctan\left(cx^2\right)\right)^2}{(ex + d)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))^2/(e*x+d)^2,x)

[Out] int((a+b*arctan(c*x^2))^2/(e*x+d)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/(e*x+d)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \arctan\left(cx^2\right)^2 + 2\, ab \arctan\left(cx^2\right) + a^2}{e^2x^2 + 2\, dex + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2)/(e^2*x^2 + 2*d*e
*x + d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))**2/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \arctan\left(cx^2\right) + a\right)^2}{(ex + d)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^2/(e*x + d)^2, x)
```

3.28 $\int (d+ex)^2\left(a+b\tan^{-1}\left(cx^3\right)\right)\,dx$

Optimal. Leaf size=315

$$\frac{(d+ex)^3\left(a+b\tan^{-1}\left(cx^3\right)\right)}{3e}+\frac{bd^2\log\left(c^{2/3}x^2+1\right)}{2\sqrt[3]{c}}-\frac{bd^2\log\left(c^{4/3}x^4-c^{2/3}x^2+1\right)}{4\sqrt[3]{c}}+\frac{\sqrt{3}bd^2\tan^{-1}\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{2\sqrt[3]{c}}-\frac{\sqrt{3}bde\log}{}$$

```
[Out] -((b*d*e*ArcTan[c^(1/3)*x])/c^(2/3)) - (b*d^3*ArcTan[c*x^3])/(3*e) + ((d +
e*x)^3*(a + b*ArcTan[c*x^3]))/(3*e) + (b*d*e*ArcTan[Sqrt[3] - 2*c^(1/3)*x])
/(2*c^(2/3)) - (b*d*e*ArcTan[Sqrt[3] + 2*c^(1/3)*x])/(2*c^(2/3)) + (Sqrt[3]
*b*d^2*ArcTan[(1 - 2*c^(2/3)*x^2)/Sqrt[3]])/(2*c^(1/3)) + (b*d^2*Log[1 + c^
(2/3)*x^2])/(2*c^(1/3)) - (Sqrt[3]*b*d*e*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3
)*x^2])/(4*c^(2/3)) + (Sqrt[3]*b*d*e*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^
2])/(4*c^(2/3)) - (b*d^2*Log[1 - c^(2/3)*x^2 + c^(4/3)*x^4])/(4*c^(1/3)) -
(b*e^2*Log[1 + c^2*x^6])/(6*c)
```

Rubi [A] time = 0.709488, antiderivative size = 331, normalized size of antiderivative = 1.05, number of steps used = 25, number of rules used = 14, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {6742, 5027, 275, 292, 31, 634, 617, 204, 628, 5033, 295, 618, 203, 260}

$$\frac{a(d+ex)^3}{3e}+\frac{bd^2\log\left(c^{2/3}x^2+1\right)}{2\sqrt[3]{c}}-\frac{bd^2\log\left(c^{4/3}x^4-c^{2/3}x^2+1\right)}{4\sqrt[3]{c}}+\frac{\sqrt{3}bd^2\tan^{-1}\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{2\sqrt[3]{c}}-\frac{\sqrt{3}bde\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x\right.}{4c^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] (a*(d + e*x)^3)/(3*e) - (b*d*e*ArcTan[c^(1/3)*x])/c^(2/3) + b*d^2*x*ArcTan[
c*x^3] + b*d*e*x^2*ArcTan[c*x^3] + (b*e^2*x^3*ArcTan[c*x^3])/3 + (b*d*e*Arc
Tan[Sqrt[3] - 2*c^(1/3)*x])/(2*c^(2/3)) - (b*d*e*ArcTan[Sqrt[3] + 2*c^(1/3)
*x])/(2*c^(2/3)) + (Sqrt[3]*b*d^2*ArcTan[(1 - 2*c^(2/3)*x^2)/Sqrt[3]])/(2*c
^(1/3)) + (b*d^2*Log[1 + c^(2/3)*x^2])/(2*c^(1/3)) - (Sqrt[3]*b*d*e*Log[1 -
 Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(4*c^(2/3)) + (Sqrt[3]*b*d*e*Log[1 + Sqr
t[3]*c^(1/3)*x + c^(2/3)*x^2])/(4*c^(2/3)) - (b*d^2*Log[1 - c^(2/3)*x^2 + c
^(4/3)*x^4])/(4*c^(1/3)) - (b*e^2*Log[1 + c^2*x^6])/(6*c)
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 5027

```
Int[ArcTan[(c_.)*(x_)^(n_)], x_Symbol] :> Simp[x*ArcTan[c*x^n], x] - Dist[c
*n, Int[x^n/(1 + c^2*x^(2*n)), x], x] /; FreeQ[{c, n}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int (d+ex)^2\left(a+b\tan^{-1}\left(cx^3\right)\right)\,dx=\int \left(a(d+ex)^2+b(d+ex)^2\tan^{-1}\left(cx^3\right)\right)\,dx$$
$$=\frac{a(d+ex)^3}{3e}+b\int (d+ex)^2\tan^{-1}\left(cx^3\right)\,dx$$
$$=\frac{a(d+ex)^3}{3e}+b\int \left(d^2\tan^{-1}\left(cx^3\right)+2dex\tan^{-1}\left(cx^3\right)+e^2x^2\tan^{-1}\left(cx^3\right)\right)\,dx$$
$$=\frac{a(d+ex)^3}{3e}+\left(bd^2\right)\int \tan^{-1}\left(cx^3\right)\,dx+(2bde)\int x\tan^{-1}\left(cx^3\right)\,dx+\left(be^2\right)\int x^2\,\text{ta}$$
$$=\frac{a(d+ex)^3}{3e}+bd^2x\tan^{-1}\left(cx^3\right)+bdex^2\tan^{-1}\left(cx^3\right)+\frac{1}{3}be^2x^3\tan^{-1}\left(cx^3\right)-\left(3bcd^2\right.$$
$$=\frac{a(d+ex)^3}{3e}+bd^2x\tan^{-1}\left(cx^3\right)+bdex^2\tan^{-1}\left(cx^3\right)+\frac{1}{3}be^2x^3\tan^{-1}\left(cx^3\right)-\frac{be^2\log}{}$$
$$=\frac{a(d+ex)^3}{3e}-\frac{bde\tan^{-1}\left(\sqrt[3]{c}x\right)}{c^{2/3}}+bd^2x\tan^{-1}\left(cx^3\right)+bdex^2\tan^{-1}\left(cx^3\right)+\frac{1}{3}be^2x^3\,\text{ta}$$
$$=\frac{a(d+ex)^3}{3e}-\frac{bde\tan^{-1}\left(\sqrt[3]{c}x\right)}{c^{2/3}}+bd^2x\tan^{-1}\left(cx^3\right)+bdex^2\tan^{-1}\left(cx^3\right)+\frac{1}{3}be^2x^3\,\text{ta}$$
$$=\frac{a(d+ex)^3}{3e}-\frac{bde\tan^{-1}\left(\sqrt[3]{c}x\right)}{c^{2/3}}+bd^2x\tan^{-1}\left(cx^3\right)+bdex^2\tan^{-1}\left(cx^3\right)+\frac{1}{3}be^2x^3\,\text{ta}$$
$$=\frac{a(d+ex)^3}{3e}-\frac{bde\tan^{-1}\left(\sqrt[3]{c}x\right)}{c^{2/3}}+bd^2x\tan^{-1}\left(cx^3\right)+bdex^2\tan^{-1}\left(cx^3\right)+\frac{1}{3}be^2x^3\,\text{ta}$$

Mathematica [A] time = 143.84, size = 297, normalized size = 0.94

$$\frac{12acd^2x+12acdex^2+4ace^2x^3+6bc^{2/3}d^2\log\left(c^{2/3}x^2+1\right)-3b\sqrt[3]{c}d\left(\sqrt[3]{c}d+\sqrt{3}e\right)\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)-3b\sqrt[3]{c}d\left(\sqrt[3]{c}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] (12*a*c*d^2*x + 12*a*c*d*e*x^2 + 4*a*c*e^2*x^3 - 12*b*c^(1/3)*d*e*ArcTan[c^
(1/3)*x] + 4*b*c*x*(3*d^2 + 3*d*e*x + e^2*x^2)*ArcTan[c*x^3] + 6*b*c^(1/3)*
d*(Sqrt[3]*c^(1/3)*d + e)*ArcTan[Sqrt[3] - 2*c^(1/3)*x] + 6*b*c^(1/3)*d*(Sq
rt[3]*c^(1/3)*d - e)*ArcTan[Sqrt[3] + 2*c^(1/3)*x] + 6*b*c^(2/3)*d^2*Log[1
+ c^(2/3)*x^2] - 3*b*c^(1/3)*d*(c^(1/3)*d + Sqrt[3]*e)*Log[1 - Sqrt[3]*c^(1
/3)*x + c^(2/3)*x^2] - 3*b*c^(1/3)*d*(c^(1/3)*d - Sqrt[3]*e)*Log[1 + Sqrt[3
```

```
]*c^(1/3)*x + c^(2/3)*x^2] - 2*b*e^2*Log[1 + c^2*x^6])/(12*c)
```

Maple [B] time = 0.095, size = 536, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(a+b*arctan(c*x^3)),x)
```

```
[Out] 1/3*a*e^2*x^3+a*e*x^2*d+a*x*d^2+1/3*a/e*d^3+1/3*b*e^2*arctan(c*x^3)*x^3+b*e
*arctan(c*x^3)*x^2*d+b*arctan(c*x^3)*x*d^2+1/3*b*d^3*arctan(c*x^3)/e+1/4*b*
e*c*ln(x^2+3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))*3^(1/2)*(1/c^2)^(5/6)*d-1
/4*b*c*ln(x^2+3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))*(1/c^2)^(2/3)*d^2-1/6*
b*e^2/c*ln(x^2+3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))-1/2*b*e/c/(1/c^2)^(1/
6)*arctan(2*x/(1/c^2)^(1/6)+3^(1/2))*d+1/2*b*c*(1/c^2)^(2/3)*arctan(2*x/(1/
c^2)^(1/6)+3^(1/2))*3^(1/2)*d^2-1/3*b/e*c*(1/c^2)^(1/2)*arctan(2*x/(1/c^2)^
(1/6)+3^(1/2))*d^3-1/4*b*e*c*ln(x^2-3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))*
3^(1/2)*(1/c^2)^(5/6)*d-1/4*b*c*ln(x^2-3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3
))*(1/c^2)^(2/3)*d^2-1/6*b*e^2/c*ln(x^2-3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/
3))-1/2*b*e/c/(1/c^2)^(1/6)*arctan(2*x/(1/c^2)^(1/6)-3^(1/2))*d-1/2*b*c*(1/
c^2)^(2/3)*arctan(2*x/(1/c^2)^(1/6)-3^(1/2))*3^(1/2)*d^2-1/3*b/e*c*(1/c^2)^
(1/2)*arctan(2*x/(1/c^2)^(1/6)-3^(1/2))*d^3+1/2*b*c*ln(x^2+(1/c^2)^(1/3))*(
1/c^2)^(2/3)*d^2-1/6*b*e^2/c*ln(x^2+(1/c^2)^(1/3))+1/3*b/e*c*(1/c^2)^(1/2)*
arctan(x/(1/c^2)^(1/6))*d^3-b*e/c/(1/c^2)^(1/6)*arctan(x/(1/c^2)^(1/6))*d
```

Maxima [A] time = 1.45915, size = 448, normalized size = 1.42

$$\frac{1}{3}ae^2x^3+adex^2-\frac{1}{4}\left(c\left(\frac{2\sqrt{3}\left(c^2\right)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(c^2\right)^{\frac{1}{3}}\left(2x^2-\frac{1}{c^2}^{\frac{1}{3}}\right)\right)}{c^2}+\frac{\left(c^2\right)^{\frac{1}{3}}\log\left(x^4-\frac{1}{c^2}^{\frac{1}{3}}x^2+\frac{1}{c^2}^{\frac{2}{3}}\right)}{c^2}-\frac{2\left(c^2\right)^{\frac{1}{3}}\log\left(x^2+\frac{1}{c^2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/3*a*e^2*x^3 + a*d*e*x^2 - 1/4*(c*(2*sqrt(3)*(c^2)^(1/3)*arctan(1/3*sqrt(3
)*(c^2)^(1/3)*(2*x^2 - (c^(-2))^(1/3)))/c^2 + (c^2)^(1/3)*log(x^4 - (c^(-2)
)^(1/3)*x^2 + (c^(-2))^(2/3))/c^2 - 2*(c^2)^(1/3)*log(x^2 + (c^(-2))^(1/3))
/c^2) - 4*x*arctan(c*x^3))*b*d^2 + 1/4*(4*x^2*arctan(c*x^3) + c*(sqrt(3)*lo
g((c^2)^(1/3)*x^2 + sqrt(3)*(c^2)^(1/6)*x + 1)/(c^2)^(5/6) - sqrt(3)*log((c
^2)^(1/3)*x^2 - sqrt(3)*(c^2)^(1/6)*x + 1)/(c^2)^(5/6) - 4*arctan((c^2)^(1/
6)*x)/(c^2)^(5/6) - 2*(c^2)^(1/6)*arctan((2*(c^2)^(1/3)*x + sqrt(3)*(c^2)^(
1/6))/(c^2)^(1/6))/c^2 - 2*(c^2)^(1/6)*arctan((2*(c^2)^(1/3)*x - sqrt(3)*(c
^2)^(1/6))/(c^2)^(1/6))/c^2))*b*d*e + a*d^2*x + 1/6*(2*c*x^3*arctan(c*x^3)
- log(c^2*x^6 + 1))*b*e^2/c
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 113.241, size = 151, normalized size = 0.48

$$ad^2x + adex^2 + \frac{ae^2x^3}{3} - 3bcd^2\,\mathrm{RootSum}\left(216t^3c^4 + 1, \left(t \mapsto t\log\left(36t^2c^2 + x^2\right)\right)\right) - 3bcde\,\mathrm{RootSum}\left(46656t^6c^{10} + 1, \left(t\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(a+b*atan(c*x**3)),x)
```

```
[Out] a*d**2*x + a*d*e*x**2 + a*e**2*x**3/3 - 3*b*c*d**2*RootSum(216*_t**3*c**4 +
 1, Lambda(_t, _t*log(36*_t**2*c**2 + x**2))) - 3*b*c*d*e*RootSum(46656*_t*
*6*c**10 + 1, Lambda(_t, _t*log(7776*_t**5*c**8 + x))) + b*d**2*x*atan(c*x*
*3) + b*d*e*x**2*atan(c*x**3) + b*e**2*Piecewise((0, Eq(c, 0)), (x**3*atan(
c*x**3)/3 - log(c**2*x**6 + 1)/(6*c), True))
```

Giac [A] time = 4.89821, size = 436, normalized size = 1.38

$$\frac{1}{4}bc^5d\left(\frac{\sqrt{3}|c|^{\frac{1}{3}}\log\left(x^2+\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^6}-\frac{\sqrt{3}|c|^{\frac{1}{3}}\log\left(x^2-\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^6}-\frac{2|c|^{\frac{1}{3}}\arctan\left(\left(2x+\frac{\sqrt{3}}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{c^6}-\frac{2|c|^{\frac{1}{3}}\arctan\left(\left(2x\right.\right.}{c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/4*b*c^5*d*(sqrt(3)*abs(c)^(1/3)*log(x^2 + sqrt(3)*x/abs(c)^(1/3) + 1/abs(
c)^(2/3))/c^6 - sqrt(3)*abs(c)^(1/3)*log(x^2 - sqrt(3)*x/abs(c)^(1/3) + 1/a
bs(c)^(2/3))/c^6 - 2*abs(c)^(1/3)*arctan((2*x + sqrt(3)/abs(c)^(1/3))*abs(c
)^(1/3))/c^6 - 2*abs(c)^(1/3)*arctan((2*x - sqrt(3)/abs(c)^(1/3))*abs(c)^(1
/3))/c^6 - 4*abs(c)^(1/3)*arctan(x*abs(c)^(1/3))/c^6)*e - 1/4*b*c^3*d^2*(2*
sqrt(3)*abs(c)^(2/3)*arctan(1/3*sqrt(3)*(2*x^2 - 1/abs(c)^(2/3))*abs(c)^(2/
3))/c^4 + abs(c)^(2/3)*log(x^4 - x^2/abs(c)^(2/3) + 1/abs(c)^(4/3))/c^4 - 2
*log(x^2 + 1/abs(c)^(2/3))/(c^2*abs(c)^(4/3))) + 1/6*(2*b*c*x^3*arctan(c*x^
3)*e^2 + 6*b*c*d*x^2*arctan(c*x^3)*e + 6*b*c*d^2*x*arctan(c*x^3) + 2*a*c*x^
3*e^2 + 6*a*c*d*x^2*e + 6*a*c*d^2*x - b*e^2*log(c^2*x^6 + 1))/c
```

3.29 $\int (d+ex)\left(a+b\tan^{-1}\left(cx^3\right)\right)\,dx$

Optimal. Leaf size=285

$$\frac{(d+ex)^2\left(a+b\tan^{-1}\left(cx^3\right)\right)}{2e}+\frac{bd\log\left(c^{2/3}x^2+1\right)}{2\sqrt[3]{c}}-\frac{bd\log\left(c^{4/3}x^4-c^{2/3}x^2+1\right)}{4\sqrt[3]{c}}+\frac{\sqrt{3}bd\tan^{-1}\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{2\sqrt[3]{c}}-\frac{\sqrt{3}be\log\left(}{}$$

```
[Out] -(b*e*ArcTan[c^(1/3)*x])/(2*c^(2/3)) - (b*d^2*ArcTan[c*x^3])/(2*e) + ((d +
e*x)^2*(a + b*ArcTan[c*x^3]))/(2*e) + (b*e*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/(
4*c^(2/3)) - (b*e*ArcTan[Sqrt[3] + 2*c^(1/3)*x])/(4*c^(2/3)) + (Sqrt[3]*b*d
*ArcTan[(1 - 2*c^(2/3)*x^2)/Sqrt[3]])/(2*c^(1/3)) + (b*d*Log[1 + c^(2/3)*x^
2])/(2*c^(1/3)) - (Sqrt[3]*b*e*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(8
*c^(2/3)) + (Sqrt[3]*b*e*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(8*c^(2/
3)) - (b*d*Log[1 - c^(2/3)*x^2 + c^(4/3)*x^4])/(4*c^(1/3))
```

Rubi [A] time = 0.60264, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 13, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.812, Rules used = {5205, 12, 1831, 275, 203, 292, 31, 634, 617, 204, 628, 295, 618}

$$\frac{(d+ex)^2\left(a+b\tan^{-1}\left(cx^3\right)\right)}{2e}+\frac{bd\log\left(c^{2/3}x^2+1\right)}{2\sqrt[3]{c}}-\frac{bd\log\left(c^{4/3}x^4-c^{2/3}x^2+1\right)}{4\sqrt[3]{c}}+\frac{\sqrt{3}bd\tan^{-1}\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{2\sqrt[3]{c}}-\frac{\sqrt{3}be\log\left(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] -(b*e*ArcTan[c^(1/3)*x])/(2*c^(2/3)) - (b*d^2*ArcTan[c*x^3])/(2*e) + ((d +
e*x)^2*(a + b*ArcTan[c*x^3]))/(2*e) + (b*e*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/(
4*c^(2/3)) - (b*e*ArcTan[Sqrt[3] + 2*c^(1/3)*x])/(4*c^(2/3)) + (Sqrt[3]*b*d
*ArcTan[(1 - 2*c^(2/3)*x^2)/Sqrt[3]])/(2*c^(1/3)) + (b*d*Log[1 + c^(2/3)*x^
2])/(2*c^(1/3)) - (Sqrt[3]*b*e*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(8
*c^(2/3)) + (Sqrt[3]*b*e*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(8*c^(2/
3)) - (b*d*Log[1 - c^(2/3)*x^2 + c^(4/3)*x^4])/(4*c^(1/3))
```

Rule 5205

```
Int[((a_.) + ArcTan[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((c + d*x)^(m + 1)*(a + b*ArcTan[u]))/(d*(m + 1)), x] - Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 + u^2), x], x], x]
/; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&
```

```
  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m +
1, x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1831

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[((c*x)^(m + ii)*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
))/(c^ii*(a + b*x^n)), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int (d+ex)\left(a+b\tan^{-1}\left(cx^3\right)\right)dx &= \frac{(d+ex)^2\left(a+b\tan^{-1}\left(cx^3\right)\right)}{2e} - \frac{b\int \frac{3cx^2(d+ex)^2}{1+c^2x^6}\,dx}{2e} \\
&= \frac{(d+ex)^2\left(a+b\tan^{-1}\left(cx^3\right)\right)}{2e} - \frac{(3bc)\int \frac{x^2(d+ex)^2}{1+c^2x^6}\,dx}{2e} \\
&= \frac{(d+ex)^2\left(a+b\tan^{-1}\left(cx^3\right)\right)}{2e} - \frac{(3bc)\int \left(\frac{d^2x^2}{1+c^2x^6} + \frac{2dex^3}{1+c^2x^6} + \frac{e^2x^4}{1+c^2x^6}\right)dx}{2e} \\
&= \frac{(d+ex)^2\left(a+b\tan^{-1}\left(cx^3\right)\right)}{2e} - (3bcd)\int \frac{x^3}{1+c^2x^6}\,dx - \frac{\left(3bcd^2\right)\int \frac{x^2}{1+c^2x^6}\,dx}{2e} - \frac{1}{2}(3bce \\
&= \frac{(d+ex)^2\left(a+b\tan^{-1}\left(cx^3\right)\right)}{2e} - \frac{1}{2}(3bcd)\,\text{Subst}\left(\int \frac{x}{1+c^2x^3}\,dx, x, x^2\right) - \frac{\left(bcd^2\right)\text{Subst} \\
&= -\frac{be\tan^{-1}\left(\sqrt[3]{c}x\right)}{2c^{2/3}} - \frac{bd^2\tan^{-1}\left(cx^3\right)}{2e} + \frac{(d+ex)^2\left(a+b\tan^{-1}\left(cx^3\right)\right)}{2e} + \frac{1}{2}\left(b\sqrt[3]{c}d\right)\text{Sub} \\
&= -\frac{be\tan^{-1}\left(\sqrt[3]{c}x\right)}{2c^{2/3}} - \frac{bd^2\tan^{-1}\left(cx^3\right)}{2e} + \frac{(d+ex)^2\left(a+b\tan^{-1}\left(cx^3\right)\right)}{2e} + \frac{bd\log\left(1+c^{2/3} \\
&= -\frac{be\tan^{-1}\left(\sqrt[3]{c}x\right)}{2c^{2/3}} - \frac{bd^2\tan^{-1}\left(cx^3\right)}{2e} + \frac{(d+ex)^2\left(a+b\tan^{-1}\left(cx^3\right)\right)}{2e} + \frac{be\tan^{-1}\left(\sqrt{3}-\right.} \\
&= -\frac{be\tan^{-1}\left(\sqrt[3]{c}x\right)}{2c^{2/3}} - \frac{bd^2\tan^{-1}\left(cx^3\right)}{2e} + \frac{(d+ex)^2\left(a+b\tan^{-1}\left(cx^3\right)\right)}{2e} + \frac{be\tan^{-1}\left(\sqrt{3}-\right.}
\end{aligned}$$

Mathematica [A] time = 0.0893988, size = 310, normalized size = 1.09

$$adx + \frac{1}{2}aex^2 - \frac{bd\left(-2\log\left(c^{2/3}x^2+1\right) + \log\left(c^{2/3}x^2 - \sqrt{3}\sqrt[3]{c}x + 1\right) + \log\left(c^{2/3}x^2 + \sqrt{3}\sqrt[3]{c}x + 1\right) - 2\sqrt{3}\tan^{-1}\left(\sqrt{3} - 2\sqrt[3]{c}x\right)\right.}{4\sqrt[3]{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 - (b*e*ArcTan[c^(1/3)*x])/(2*c^(2/3)) + b*d*x*ArcTan[c*
x^3] + (b*e*x^2*ArcTan[c*x^3])/2 + (b*e*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/(4*c
^(2/3)) - (b*e*ArcTan[Sqrt[3] + 2*c^(1/3)*x])/(4*c^(2/3)) - (Sqrt[3]*b*e*Lo
g[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(8*c^(2/3)) + (Sqrt[3]*b*e*Log[1 +
Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(8*c^(2/3)) - (b*d*(-2*Sqrt[3]*ArcTan[Sqr
```

```
t[3] - 2*c^(1/3)*x] - 2*Sqrt[3]*ArcTan[Sqrt[3] + 2*c^(1/3)*x] - 2*Log[1 + c
^(2/3)*x^2] + Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2] + Log[1 + Sqrt[3]*c^
(1/3)*x + c^(2/3)*x^2]))/(4*c^(1/3))
```

Maple [A] time = 0.069, size = 314, normalized size = 1.1

$$\frac{ax^2e}{2} + adx + \frac{b\arctan\left(cx^3\right)x^2e}{2} + b\arctan\left(cx^3\right)dx + \frac{bc\sqrt{3}e}{8}\ln\left(x^2 + \sqrt{3}\sqrt[6]{c^{-2}}x + \sqrt[3]{c^{-2}}\right)\left(c^{-2}\right)^{\frac{5}{6}} - \frac{bcd}{4}\ln\left(x^2 + \sqrt{3}\sqrt[6]{c^{-2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(a+b*arctan(c*x^3)),x)
```

```
[Out] 1/2*a*x^2*e+a*d*x+1/2*b*arctan(c*x^3)*x^2*e+b*arctan(c*x^3)*d*x+1/8*b*c*ln(
x^2+3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))*3^(1/2)*(1/c^2)^(5/6)*e-1/4*b*c*
ln(x^2+3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))*(1/c^2)^(2/3)*d-1/4*b/c/(1/c^
2)^(1/6)*arctan(2*x/(1/c^2)^(1/6)+3^(1/2))*e+1/2*b*c*(1/c^2)^(2/3)*arctan(2
*x/(1/c^2)^(1/6)+3^(1/2))*3^(1/2)*d-1/4*b*c^3*ln(x^2-3^(1/2)*(1/c^2)^(1/6)*
x+(1/c^2)^(1/3))*(1/c^2)^(5/3)*d-1/8*b*c*ln(x^2-3^(1/2)*(1/c^2)^(1/6)*x+(1/
c^2)^(1/3))*3^(1/2)*(1/c^2)^(5/6)*e-1/2*b*c^3*(1/c^2)^(5/3)*arctan(2*x/(1/c
^2)^(1/6)-3^(1/2))*3^(1/2)*d-1/4*b/c/(1/c^2)^(1/6)*arctan(2*x/(1/c^2)^(1/6)
-3^(1/2))*e+1/2*b*c*(1/c^2)^(2/3)*d*ln(x^2+(1/c^2)^(1/3))-1/2*b/c*e/(1/c^2)
^(1/6)*arctan(x/(1/c^2)^(1/6))
```

Maxima [A] time = 1.47298, size = 383, normalized size = 1.34

$$\frac{1}{2}aex^2 - \frac{1}{4}\left(c\left(\frac{2\sqrt{3}\left(c^2\right)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(c^2\right)^{\frac{1}{3}}\left(2x^2 - \frac{1}{c^2}^{\frac{1}{3}}\right)\right)}{c^2} + \frac{\left(c^2\right)^{\frac{1}{3}}\log\left(x^4 - \frac{1}{c^2}^{\frac{1}{3}}x^2 + \frac{1}{c^2}^{\frac{2}{3}}\right)}{c^2} - \frac{2\left(c^2\right)^{\frac{1}{3}}\log\left(x^2 + \frac{1}{c^2}^{\frac{1}{3}}\right)}{c^2}\right) - 4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/2*a*e*x^2 - 1/4*(c*(2*sqrt(3)*(c^2)^(1/3)*arctan(1/3*sqrt(3)*(c^2)^(1/3)*
(2*x^2 - (c^(-2))^(1/3)))/c^2 + (c^2)^(1/3)*log(x^4 - (c^(-2))^(1/3)*x^2 +
```

```
(c^(-2))^(2/3))/c^2 - 2*(c^2)^(1/3)*log(x^2 + (c^(-2))^(1/3))/c^2) - 4*x*ar
ctan(c*x^3))*b*d + 1/8*(4*x^2*arctan(c*x^3) + c*(sqrt(3)*log((c^2)^(1/3)*x^
2 + sqrt(3)*(c^2)^(1/6)*x + 1)/(c^2)^(5/6) - sqrt(3)*log((c^2)^(1/3)*x^2 -
sqrt(3)*(c^2)^(1/6)*x + 1)/(c^2)^(5/6) - 4*arctan((c^2)^(1/6)*x)/(c^2)^(5/6
) - 2*(c^2)^(1/6)*arctan((2*(c^2)^(1/3)*x + sqrt(3)*(c^2)^(1/6))/(c^2)^(1/6
))/c^2 - 2*(c^2)^(1/6)*arctan((2*(c^2)^(1/3)*x - sqrt(3)*(c^2)^(1/6))/(c^2)
^(1/6))/c^2))*b*e + a*d*x
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 61.3176, size = 104, normalized size = 0.36

$$adx + \frac{aex^2}{2} - 3bcd\,\mathrm{RootSum}\left(216t^3c^4 + 1, \left(t \mapsto t\log\left(36t^2c^2 + x^2\right)\right)\right) - \frac{3bce\,\mathrm{RootSum}\left(46656t^6c^{10} + 1, \left(t \mapsto t\log\left(777\right.\right.\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*atan(c*x**3)),x)
```

```
[Out] a*d*x + a*e*x**2/2 - 3*b*c*d*RootSum(216*_t**3*c**4 + 1, Lambda(_t, _t*log(
36*_t**2*c**2 + x**2))) - 3*b*c*e*RootSum(46656*_t**6*c**10 + 1, Lambda(_t,
 _t*log(7776*_t**5*c**8 + x)))/2 + b*d*x*atan(c*x**3) + b*e*x**2*atan(c*x**
3)/2
```

Giac [A] time = 2.48028, size = 355, normalized size = 1.25

$$\frac{1}{8}bc^5\left(\frac{\sqrt{3}|c|^{\frac{1}{3}}\log\left(x^2 + \frac{\sqrt{3}x}{|c|^{\frac{1}{3}}} + \frac{1}{|c|^{\frac{2}{3}}}\right)}{c^6} - \frac{\sqrt{3}|c|^{\frac{1}{3}}\log\left(x^2 - \frac{\sqrt{3}x}{|c|^{\frac{1}{3}}} + \frac{1}{|c|^{\frac{2}{3}}}\right)}{c^6} - \frac{2|c|^{\frac{1}{3}}\arctan\left(\left(2x + \frac{\sqrt{3}}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{c^6} - \frac{2|c|^{\frac{1}{3}}\arctan\left(\left(2x -\right.\right.\right.}{c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/8*b*c^5*(sqrt(3)*abs(c)^(1/3)*log(x^2 + sqrt(3)*x/abs(c)^(1/3) + 1/abs(c)
^(2/3))/c^6 - sqrt(3)*abs(c)^(1/3)*log(x^2 - sqrt(3)*x/abs(c)^(1/3) + 1/abs
(c)^(2/3))/c^6 - 2*abs(c)^(1/3)*arctan((2*x + sqrt(3)/abs(c)^(1/3))*abs(c)^
(1/3))/c^6 - 2*abs(c)^(1/3)*arctan((2*x - sqrt(3)/abs(c)^(1/3))*abs(c)^(1/3
))/c^6 - 4*abs(c)^(1/3)*arctan(x*abs(c)^(1/3))/c^6)*e - 1/4*b*c^3*d*(2*sqrt
(3)*abs(c)^(2/3)*arctan(1/3*sqrt(3)*(2*x^2 - 1/abs(c)^(2/3))*abs(c)^(2/3))/
c^4 + abs(c)^(2/3)*log(x^4 - x^2/abs(c)^(2/3) + 1/abs(c)^(4/3))/c^4 - 2*log
(x^2 + 1/abs(c)^(2/3))/(c^2*abs(c)^(4/3))) + 1/2*b*x^2*arctan(c*x^3)*e + b*
d*x*arctan(c*x^3) + 1/2*a*x^2*e + a*d*x
```

3.30 $\int \frac{a+b\tan^{-1}\left(cx^3\right)}{d+ex}\,dx$

Optimal. Leaf size=739

$$-\frac{bc\text{PolyLog}\left(2,\frac{\sqrt[6]{-c^2}(d+ex)}{\sqrt[6]{-c^2}d-e}\right)}{2\sqrt{-c^2}e}+\frac{bc\text{PolyLog}\left(2,\frac{\sqrt[6]{-c^2}(d+ex)}{\sqrt[6]{-c^2}d+e}\right)}{2\sqrt{-c^2}e}+\frac{bc\text{PolyLog}\left(2,\frac{\sqrt[6]{-c^2}(d+ex)}{\sqrt[6]{-c^2}d-\sqrt[3]{-1}e}\right)}{2\sqrt{-c^2}e}-\frac{bc\text{PolyLog}\left(2,\frac{\sqrt[6]{-c^2}(d+ex)}{\sqrt[6]{-c^2}d+\sqrt[3]{-1}e}\right)}{2\sqrt{-c^2}e}-$$

```
[Out] ((a + b*ArcTan[c*x^3])*Log[d + e*x])/e + (b*c*Log[(e*(1 - (-c^2)^(1/6)*x))/
((-c^2)^(1/6)*d + e)]*Log[d + e*x])/(2*Sqrt[-c^2]*e) - (b*c*Log[-((e*(1 + (
-c^2)^(1/6)*x))/((-c^2)^(1/6)*d - e))]*Log[d + e*x])/(2*Sqrt[-c^2]*e) + (b*
c*Log[-((e*((-1)^(1/3) + (-c^2)^(1/6)*x))/((-c^2)^(1/6)*d - (-1)^(1/3)*e))]
*Log[d + e*x])/(2*Sqrt[-c^2]*e) - (b*c*Log[-((e*((-1)^(2/3) + (-c^2)^(1/6)*
x))/((-c^2)^(1/6)*d - (-1)^(2/3)*e))]*Log[d + e*x])/(2*Sqrt[-c^2]*e) + (b*c
*Log[((-1)^(2/3)*e*(1 + (-1)^(1/3)*(-c^2)^(1/6)*x))/((-c^2)^(1/6)*d + (-1)^
(2/3)*e)]*Log[d + e*x])/(2*Sqrt[-c^2]*e) - (b*c*Log[((-1)^(1/3)*e*(1 + (-1)
^(2/3)*(-c^2)^(1/6)*x))/((-c^2)^(1/6)*d + (-1)^(1/3)*e)]*Log[d + e*x])/(2*S
qrt[-c^2]*e) - (b*c*PolyLog[2, ((-c^2)^(1/6)*(d + e*x))/((-c^2)^(1/6)*d - e
)])/(2*Sqrt[-c^2]*e) + (b*c*PolyLog[2, ((-c^2)^(1/6)*(d + e*x))/((-c^2)^(1/
6)*d + e)])/(2*Sqrt[-c^2]*e) + (b*c*PolyLog[2, ((-c^2)^(1/6)*(d + e*x))/((-
c^2)^(1/6)*d - (-1)^(1/3)*e)])/(2*Sqrt[-c^2]*e) - (b*c*PolyLog[2, ((-c^2)^(
1/6)*(d + e*x))/((-c^2)^(1/6)*d + (-1)^(1/3)*e)])/(2*Sqrt[-c^2]*e) - (b*c*P
olyLog[2, ((-c^2)^(1/6)*(d + e*x))/((-c^2)^(1/6)*d - (-1)^(2/3)*e)])/(2*Sqr
t[-c^2]*e) + (b*c*PolyLog[2, ((-c^2)^(1/6)*(d + e*x))/((-c^2)^(1/6)*d + (-1
)^(2/3)*e)])/(2*Sqrt[-c^2]*e)
```

Rubi [F] time = 0.0625405, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{a+b\tan^{-1}\left(cx^3\right)}{d+ex}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTan[c*x^3])/(d + e*x),x]
```

```
[Out] (a*Log[d + e*x])/e + b*Defer[Int][ArcTan[c*x^3]/(d + e*x), x]
```

Rubi steps

$$\int \frac{a + b\tan^{-1}\left(cx^3\right)}{d + ex}\,dx = \int \left(\frac{a}{d + ex} + \frac{b\tan^{-1}\left(cx^3\right)}{d + ex}\right) dx$$
$$= \frac{a\log(d + ex)}{e} + b\int \frac{\tan^{-1}\left(cx^3\right)}{d + ex}\,dx$$

Mathematica [F] time = 180.003, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x^3])/(d + e*x),x]

[Out] $Aborted
```

Maple [C] time = 0.128, size = 172, normalized size = 0.2

$$\frac{a\ln(ex + d)}{e} + \frac{b\ln(ex + d)\arctan\left(cx^3\right)}{e} - \frac{be^2}{2c}\sum_{\_R1=\mathrm{RootOf}\left(\_Z^6c^2-6c^2d\_Z^5+15c^2d^2\_Z^4-20c^2d^3\_Z^3+15c^2d^4\_Z^2-6c^2d^5\_Z+c^2d^6+e^6\right)} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))/(e*x+d),x)

[Out] a*ln(e*x+d)/e+b*ln(e*x+d)/e*arctan(c*x^3)-1/2*b*e^2/c*sum(1/(_R1^3-3*_R1^2*
d+3*_R1*d^2-d^3)*(ln(e*x+d)*ln((-e*x+_R1-d)/_R1)+dilog((-e*x+_R1-d)/_R1)),_
R1=RootOf(_Z^6*c^2-6*_Z^5*c^2*d+15*_Z^4*c^2*d^2-20*_Z^3*c^2*d^3+15*_Z^2*c^2
*d^4-6*_Z*c^2*d^5+c^2*d^6+e^6))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$2b\int \frac{\arctan\left(cx^3\right)}{2(ex + d)}\,dx + \frac{a\log(ex + d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/(e*x+d),x, algorithm="maxima")
```

```
[Out] 2*b*integrate(1/2*arctan(c*x^3)/(e*x + d), x) + a*log(e*x + d)/e
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\arctan\left(cx^3\right)+a}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x^3) + a)/(e*x + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\arctan\left(cx^3\right)+a}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^3) + a)/(e*x + d), x)
```

3.31 $\int \frac{a+b\tan^{-1}\left(cx^3\right)}{(d+ex)^2}\,dx$

Optimal. Leaf size=906

$$\frac{bc^2\tan^{-1}\left(cx^3\right)d^5}{e\left(c^2d^6+e^6\right)}+\frac{bc^{5/3}\log\left(c^{2/3}x^2+1\right)d^4}{2\left(c^2d^6+e^6\right)}+\frac{3bce^2\log(d+ex)d^2}{c^2d^6+e^6}-\frac{bce^2\log\left(c^2x^6+1\right)d^2}{2\left(c^2d^6+e^6\right)}-\frac{bc^{2/3}e^3\tan^{-1}\left(\sqrt[3]{c}x\right)d}{c^2d^6+e^6}+$$

```
[Out] -((b*c^(2/3)*d*e^3*ArcTan[c^(1/3)*x])/(c^2*d^6 + e^6)) + (b*c^2*d^5*ArcTan[
c*x^3])/(e*(c^2*d^6 + e^6)) - (a + b*ArcTan[c*x^3])/(e*(d + e*x)) + (b*c^(2
/3)*d*(Sqrt[3]*c*d^3 + e^3)*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/(2*(c^2*d^6 + e^
6)) + (b*c^(2/3)*d*(Sqrt[3]*c*d^3 - e^3)*ArcTan[Sqrt[3] + 2*c^(1/3)*x])/(2*
(c^2*d^6 + e^6)) + (Sqrt[3]*b*c^(5/3)*e*(Sqrt[-c^2]*d^3 + e^3)*ArcTan[(1 +
(2*c^(2/3)*x)/(-c^2)^(1/6))/Sqrt[3]])/(2*(-c^2)^(2/3)*(c^2*d^6 + e^6)) - (S
qrt[3]*b*c^(5/3)*e*(Sqrt[-c^2]*d^3 - e^3)*ArcTan[(c^(4/3) + 2*(-c^2)^(5/6)*
x)/(Sqrt[3]*c^(4/3))])/(2*(-c^2)^(2/3)*(c^2*d^6 + e^6)) + (b*c^(5/3)*e*(Sqr
t[-c^2]*d^3 + e^3)*Log[(-c^2)^(1/6) - c^(2/3)*x])/(2*(-c^2)^(2/3)*(c^2*d^6
+ e^6)) - (b*c^(5/3)*e*(Sqrt[-c^2]*d^3 - e^3)*Log[(-c^2)^(1/6) + c^(2/3)*x]
)/(2*(-c^2)^(2/3)*(c^2*d^6 + e^6)) + (3*b*c*d^2*e^2*Log[d + e*x])/(c^2*d^6
+ e^6) + (b*c^(5/3)*d^4*Log[1 + c^(2/3)*x^2])/(2*(c^2*d^6 + e^6)) - (b*c^(2
/3)*d*(c*d^3 - Sqrt[3]*e^3)*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(4*(c
^2*d^6 + e^6)) - (b*c^(2/3)*d*(c*d^3 + Sqrt[3]*e^3)*Log[1 + Sqrt[3]*c^(1/3)
*x + c^(2/3)*x^2])/(4*(c^2*d^6 + e^6)) + (b*c^(5/3)*e*(Sqrt[-c^2]*d^3 - e^3
)*Log[(-c^2)^(1/3) - c^(2/3)*(-c^2)^(1/6)*x + c^(4/3)*x^2])/(4*(-c^2)^(2/3)
*(c^2*d^6 + e^6)) - (b*c^(5/3)*e*(Sqrt[-c^2]*d^3 + e^3)*Log[(-c^2)^(1/3) +
c^(2/3)*(-c^2)^(1/6)*x + c^(4/3)*x^2])/(4*(-c^2)^(2/3)*(c^2*d^6 + e^6)) - (
b*c*d^2*e^2*Log[1 + c^2*x^6])/(2*(c^2*d^6 + e^6))
```

Rubi [A] time = 1.4838, antiderivative size = 906, normalized size of antiderivative = 1., number of steps used = 35, number of rules used = 16, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.889, Rules used = {5205, 12, 6725, 1876, 1416, 635, 203, 260, 634, 617, 204, 628, 1511, 292, 31, 1469}

$$\frac{bc^2\tan^{-1}\left(cx^3\right)d^5}{e\left(c^2d^6+e^6\right)}+\frac{bc^{5/3}\log\left(c^{2/3}x^2+1\right)d^4}{2\left(c^2d^6+e^6\right)}+\frac{3bce^2\log(d+ex)d^2}{c^2d^6+e^6}-\frac{bce^2\log\left(c^2x^6+1\right)d^2}{2\left(c^2d^6+e^6\right)}-\frac{bc^{2/3}e^3\tan^{-1}\left(\sqrt[3]{c}x\right)d}{c^2d^6+e^6}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^3])/(d + e*x)^2,x]
```

```
[Out] -((b*c^(2/3)*d*e^3*ArcTan[c^(1/3)*x])/(c^2*d^6 + e^6)) + (b*c^2*d^5*ArcTan[
c*x^3])/(e*(c^2*d^6 + e^6)) - (a + b*ArcTan[c*x^3])/(e*(d + e*x)) + (b*c^(2
/3)*d*(Sqrt[3]*c*d^3 + e^3)*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/(2*(c^2*d^6 + e^
6)) + (b*c^(2/3)*d*(Sqrt[3]*c*d^3 - e^3)*ArcTan[Sqrt[3] + 2*c^(1/3)*x])/(2*
(c^2*d^6 + e^6)) + (Sqrt[3]*b*c^(5/3)*e*(Sqrt[-c^2]*d^3 + e^3)*ArcTan[(1 +
(2*c^(2/3)*x)/(-c^2)^(1/6))/Sqrt[3]])/(2*(-c^2)^(2/3)*(c^2*d^6 + e^6)) - (S
qrt[3]*b*c^(5/3)*e*(Sqrt[-c^2]*d^3 - e^3)*ArcTan[(c^(4/3) + 2*(-c^2)^(5/6)*
x)/(Sqrt[3]*c^(4/3))])/(2*(-c^2)^(2/3)*(c^2*d^6 + e^6)) + (b*c^(5/3)*e*(Sqr
t[-c^2]*d^3 + e^3)*Log[(-c^2)^(1/6) - c^(2/3)*x])/(2*(-c^2)^(2/3)*(c^2*d^6
+ e^6)) - (b*c^(5/3)*e*(Sqrt[-c^2]*d^3 - e^3)*Log[(-c^2)^(1/6) + c^(2/3)*x]
)/(2*(-c^2)^(2/3)*(c^2*d^6 + e^6)) + (3*b*c*d^2*e^2*Log[d + e*x])/(c^2*d^6
+ e^6) + (b*c^(5/3)*d^4*Log[1 + c^(2/3)*x^2])/(2*(c^2*d^6 + e^6)) - (b*c^(2
/3)*d*(c*d^3 - Sqrt[3]*e^3)*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(4*(c
^2*d^6 + e^6)) - (b*c^(2/3)*d*(c*d^3 + Sqrt[3]*e^3)*Log[1 + Sqrt[3]*c^(1/3)
*x + c^(2/3)*x^2])/(4*(c^2*d^6 + e^6)) + (b*c^(5/3)*e*(Sqrt[-c^2]*d^3 - e^3
)*Log[(-c^2)^(1/3) - c^(2/3)*(-c^2)^(1/6)*x + c^(4/3)*x^2])/(4*(-c^2)^(2/3)
*(c^2*d^6 + e^6)) - (b*c^(5/3)*e*(Sqrt[-c^2]*d^3 + e^3)*Log[(-c^2)^(1/3) +
c^(2/3)*(-c^2)^(1/6)*x + c^(4/3)*x^2])/(4*(-c^2)^(2/3)*(c^2*d^6 + e^6)) - (
b*c*d^2*e^2*Log[1 + c^2*x^6])/(2*(c^2*d^6 + e^6))
```

Rule 5205

```
Int[((a_.) + ArcTan[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((c + d*x)^(m + 1)*(a + b*ArcTan[u]))/(d*(m + 1)), x] - Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 + u^2), x], x], x]
/; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&
  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m +
1, x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 1416

```
Int[((d_) + (e_.)*(x_)^3)/((a_) + (c_.)*(x_)^6), x_Symbol] :> With[{q = Rt[
c/a, 6]}, Dist[1/(3*a*q^2), Int[(q^2*d - e*x)/(1 + q^2*x^2), x], x] + (Dist
[1/(6*a*q^2), Int[(2*q^2*d - (Sqrt[3]*q^3*d - e)*x)/(1 - Sqrt[3]*q*x + q^2*
x^2), x], x] + Dist[1/(6*a*q^2), Int[(2*q^2*d + (Sqrt[3]*q^3*d + e)*x)/(1 +
 Sqrt[3]*q*x + q^2*x^2), x], x])] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && PosQ[c/a]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1511

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (c_.)*(x_)^(n2_))
, x_Symbol] :> With[{q = Rt[-(a*c), 2]}, -Dist[e/2 + (c*d)/(2*q), Int[(f*x)
^m/(q - c*x^n), x], x] + Dist[e/2 - (c*d)/(2*q), Int[(f*x)^m/(q + c*x^n), x
], x]] /; FreeQ[{a, c, d, e, f, m}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 1469

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^n
], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify
[m - n + 1], 0]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}\left(cx^3\right)}{(d+ex)^2}\,dx = -\frac{a+b\tan^{-1}\left(cx^3\right)}{e(d+ex)} + \frac{b\int \frac{3cx^2}{(d+ex)\left(1+c^2x^6\right)}\,dx}{e}$$

$$= -\frac{a+b\tan^{-1}\left(cx^3\right)}{e(d+ex)} + \frac{(3bc)\int \frac{x^2}{(d+ex)\left(1+c^2x^6\right)}\,dx}{e}$$

$$= -\frac{a+b\tan^{-1}\left(cx^3\right)}{e(d+ex)} + \frac{(3bc)\int \left(\frac{d^2e^4}{\left(c^2d^6+e^6\right)(d+ex)} + \frac{(d-ex)\left(-e^4+c^2d^4x^2+c^2d^2e^2x^4\right)}{\left(c^2d^6+e^6\right)\left(1+c^2x^6\right)}\right)dx}{e}$$

$$= -\frac{a+b\tan^{-1}\left(cx^3\right)}{e(d+ex)} + \frac{3bcd^2e^2\log(d+ex)}{c^2d^6+e^6} + \frac{(3bc)\int \frac{(d-ex)\left(-e^4+c^2d^4x^2+c^2d^2e^2x^4\right)}{1+c^2x^6}\,dx}{e\left(c^2d^6+e^6\right)}$$

$$= -\frac{a+b\tan^{-1}\left(cx^3\right)}{e(d+ex)} + \frac{3bcd^2e^2\log(d+ex)}{c^2d^6+e^6} + \frac{(3bc)\int \left(\frac{-de^4-c^2d^4ex^3}{1+c^2x^6} + \frac{x\left(e^5+c^2d^3e^2x^3\right)}{1+c^2x^6} + \frac{x^2\left(c^2d^5-c^2d^2\right.}{1+c^2x^6}\right.}{e\left(c^2d^6+e^6\right)}$$

$$= -\frac{a+b\tan^{-1}\left(cx^3\right)}{e(d+ex)} + \frac{3bcd^2e^2\log(d+ex)}{c^2d^6+e^6} + \frac{(3bc)\int \frac{-de^4-c^2d^4ex^3}{1+c^2x^6}\,dx}{e\left(c^2d^6+e^6\right)} + \frac{(3bc)\int \frac{x\left(e^5+c^2d^3e^2x^3\right)}{1+c^2x^6}\,dx}{e\left(c^2d^6+e^6\right)}$$

$$= -\frac{a+b\tan^{-1}\left(cx^3\right)}{e(d+ex)} + \frac{3bcd^2e^2\log(d+ex)}{c^2d^6+e^6} + \frac{\left(b\sqrt[3]{c}\right)\int \frac{-2c^{2/3}de^4-\left(c^2d^4e-\sqrt{3}cde^4\right)x}{1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{2e\left(c^2d^6+e^6\right)} + \frac{\left(b\sqrt[3]{c}\right)\int \frac{-2c}{2}}{2}$$

$$= -\frac{a+b\tan^{-1}\left(cx^3\right)}{e(d+ex)} + \frac{3bcd^2e^2\log(d+ex)}{c^2d^6+e^6} + \frac{\left(bc^{7/3}d^4\right)\int \frac{x}{1+c^{2/3}x^2}\,dx}{c^2d^6+e^6} + \frac{\left(bc^3d^5\right)\mathrm{Subst}\left(\int \frac{1}{1+c^2x^2}\right.}{e\left(c^2d^6+e^6\right)}$$

$$= -\frac{bc^{2/3}de^3\tan^{-1}\left(\sqrt[3]{c}x\right)}{c^2d^6+e^6} + \frac{bc^2d^5\tan^{-1}\left(cx^3\right)}{e\left(c^2d^6+e^6\right)} - \frac{a+b\tan^{-1}\left(cx^3\right)}{e(d+ex)} + \frac{bc^{5/3}e\left(\sqrt{-c^2}d^3+e^3\right)\log\left(\right.}{2\left(-c^2\right)^{2/3}\left(c^2d^6-\right.}$$

$$= -\frac{bc^{2/3}de^3\tan^{-1}\left(\sqrt[3]{c}x\right)}{c^2d^6+e^6} + \frac{bc^2d^5\tan^{-1}\left(cx^3\right)}{e\left(c^2d^6+e^6\right)} - \frac{a+b\tan^{-1}\left(cx^3\right)}{e(d+ex)} + \frac{bc^{2/3}d\left(\sqrt{3}cd^3+e^3\right)\tan^{-1}}{2\left(c^2d^6+e^6\right)}$$

$$= -\frac{bc^{2/3}de^3\tan^{-1}\left(\sqrt[3]{c}x\right)}{c^2d^6+e^6} + \frac{bc^2d^5\tan^{-1}\left(cx^3\right)}{e\left(c^2d^6+e^6\right)} - \frac{a+b\tan^{-1}\left(cx^3\right)}{e(d+ex)} + \frac{bc^{2/3}d\left(\sqrt{3}cd^3+e^3\right)\tan^{-1}}{2\left(c^2d^6+e^6\right)}$$

Mathematica [A] time = 14.4625, size = 536, normalized size = 0.59

$$\frac{-4a\sqrt[3]{c}\left(c^2d^6+e^6\right)-2bc^{4/3}d^2e^3\log\left(c^2x^6+1\right)(d+ex)+2be\left(c^2d^4+c^{2/3}e^4\right)\log\left(c^{2/3}x^2+1\right)(d+ex)-bc^{2/3}e\left(c^{4/3}d^4-\sqrt{3}c$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^3])/(d + e*x)^2,x]
```

```
[Out] (-4*a*c^(1/3)*(c^2*d^6 + e^6) - 4*b*c*d*(c^(4/3)*d^4 - c^(2/3)*d^2*e^2 + e^
4)*(d + e*x)*ArcTan[c^(1/3)*x] - 4*b*c^(1/3)*(c^2*d^6 + e^6)*ArcTan[c*x^3]
- 2*b*c^(2/3)*(2*c^(5/3)*d^5 - Sqrt[3]*c^(4/3)*d^4*e + c*d^3*e^2 - c^(1/3)*
d*e^4 + Sqrt[3]*e^5)*(d + e*x)*ArcTan[Sqrt[3] - 2*c^(1/3)*x] + 2*b*c^(2/3)*
(2*c^(5/3)*d^5 + Sqrt[3]*c^(4/3)*d^4*e + c*d^3*e^2 - c^(1/3)*d*e^4 - Sqrt[3
]*e^5)*(d + e*x)*ArcTan[Sqrt[3] + 2*c^(1/3)*x] + 12*b*c^(4/3)*d^2*e^3*(d +
e*x)*Log[d + e*x] + 2*b*e*(c^2*d^4 + c^(2/3)*e^4)*(d + e*x)*Log[1 + c^(2/3)
*x^2] - b*c^(2/3)*e*(c^(4/3)*d^4 - Sqrt[3]*c*d^3*e - Sqrt[3]*c^(1/3)*d*e^3
+ e^4)*(d + e*x)*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2] - b*c^(2/3)*e*(c^
(4/3)*d^4 + Sqrt[3]*c*d^3*e + Sqrt[3]*c^(1/3)*d*e^3 + e^4)*(d + e*x)*Log[1
+ Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2] - 2*b*c^(4/3)*d^2*e^3*(d + e*x)*Log[1 +
c^2*x^6])/(4*c^(1/3)*e*(c^2*d^6 + e^6)*(d + e*x))
```

Maple [A] time = 0.139, size = 1220, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))/(e*x+d)^2,x)
```

```
[Out] 3*b*c*d^2*e^2*ln(e*x+d)/(c^2*d^6+e^6)-1/4*b*e^3*c^3/(c^2*d^6+e^6)*ln(x^2+3^
(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))*3^(1/2)*(1/c^2)^(7/6)*d-1/4*b*e*c^3/(c
^2*d^6+e^6)*ln(x^2+3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))*3^(1/2)*(1/c^2)^(
5/6)*d^3+1/4*b*e^3*c^3/(c^2*d^6+e^6)*ln(x^2-3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)
^(1/3))*3^(1/2)*(1/c^2)^(7/6)*d+1/4*b*e*c^3/(c^2*d^6+e^6)*ln(x^2-3^(1/2)*(1
/c^2)^(1/6)*x+(1/c^2)^(1/3))*3^(1/2)*(1/c^2)^(5/6)*d^3-1/4*b*c^3/(c^2*d^6+e
^6)*ln(x^2+3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))*(1/c^2)^(2/3)*d^4-1/4*b*c
^3/(c^2*d^6+e^6)*ln(x^2-3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))*(1/c^2)^(2/3
)*d^4+1/2*b*c^3/(c^2*d^6+e^6)*ln(x^2+(1/c^2)^(1/3))*(1/c^2)^(2/3)*d^4-1/2*b
*e^2*c/(c^2*d^6+e^6)*ln(x^2+(1/c^2)^(1/3))*d^2-1/4*b*e^4*c/(c^2*d^6+e^6)*ln
(x^2+3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))*(1/c^2)^(1/3)-1/2*b*e^2*c/(c^2*
```

```
d^6+e^6)*ln(x^2+3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))*d^2-1/4*b*e^4*c/(c^2
*d^6+e^6)*ln(x^2-3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))*(1/c^2)^(1/3)-1/2*b
*e^2*c/(c^2*d^6+e^6)*ln(x^2-3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))*d^2+1/2*
b*e^4*c/(c^2*d^6+e^6)*ln(x^2+(1/c^2)^(1/3))*(1/c^2)^(1/3)-a/(e*x+d)/e-1/2*b
*c^3/(c^2*d^6+e^6)*(1/c^2)^(2/3)*arctan(2*x/(1/c^2)^(1/6)-3^(1/2))*3^(1/2)*
d^4+1/2*b*c^3/(c^2*d^6+e^6)*(1/c^2)^(2/3)*arctan(2*x/(1/c^2)^(1/6)+3^(1/2))
*3^(1/2)*d^4-1/2*b*e^3*c^3/(c^2*d^6+e^6)*(1/c^2)^(7/6)*arctan(2*x/(1/c^2)^(
1/6)+3^(1/2))*d-1/2*b*e^3*c^3/(c^2*d^6+e^6)*(1/c^2)^(7/6)*arctan(2*x/(1/c^2
)^(1/6)-3^(1/2))*d+b*e^4*c^3/(c^2*d^6+e^6)*(1/c^2)^(4/3)*arctan(2*x/(1/c^2)
^(1/6)-3^(1/2))*3^(1/2)-b*e^3*c^3/(c^2*d^6+e^6)*(1/c^2)^(7/6)*arctan(x/(1/c
^2)^(1/6))*d+1/2*b*e*c/(c^2*d^6+e^6)/(1/c^2)^(1/6)*arctan(2*x/(1/c^2)^(1/6)
-3^(1/2))*d^3-b/(e*x+d)/e*arctan(c*x^3)+1/2*b*e*c/(c^2*d^6+e^6)/(1/c^2)^(1/
6)*arctan(2*x/(1/c^2)^(1/6)+3^(1/2))*d^3-1/2*b*e^4*c/(c^2*d^6+e^6)*(1/c^2)^
(1/3)*arctan(2*x/(1/c^2)^(1/6)+3^(1/2))*3^(1/2)+b/e*c^3/(c^2*d^6+e^6)*(1/c^
2)^(1/2)*arctan(2*x/(1/c^2)^(1/6)+3^(1/2))*d^5-1/2*b*e^4*c/(c^2*d^6+e^6)*(1
/c^2)^(1/3)*arctan(2*x/(1/c^2)^(1/6)-3^(1/2))*3^(1/2)+b/e*c^3/(c^2*d^6+e^6)
*(1/c^2)^(1/2)*arctan(2*x/(1/c^2)^(1/6)-3^(1/2))*d^5-b/e*c^3/(c^2*d^6+e^6)*
(1/c^2)^(1/2)*arctan(x/(1/c^2)^(1/6))*d^5+b*e*c/(c^2*d^6+e^6)/(1/c^2)^(1/6)
*arctan(x/(1/c^2)^(1/6))*d^3
```

Maxima [A] time = 1.56955, size = 1018, normalized size = 1.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/4*((12*d^2*e^2*log(e*x + d)/(c^2*d^6 + e^6) - (2*((c^2)^(1/3)*c^2*d^5 - c
^2*d^3*e^2 + (c^2)^(2/3)*d*e^4)*log(((c^2)^(1/3)*x - sqrt(-(c^2)^(1/3)))/((
c^2)^(1/3)*x + sqrt(-(c^2)^(1/3))))/((c^2)^(2/3)*sqrt(-(c^2)^(1/3))) - 2*((
c^2)^(1/3)*c^2*d^4*e - c^2*d^2*e^3 + (c^2)^(2/3)*e^5)*log((c^2)^(1/3)*x^2 +
 1)/c^2 + (sqrt(3)*sqrt(c^2)*c^2*d^3*e^2 + (c^2)^(2/3)*c^2*d^4*e + sqrt(3)*
(c^2)^(1/6)*c^2*d*e^4 + 2*(c^2)^(1/3)*c^2*d^2*e^3 + c^2*e^5)*log((c^2)^(1/3
)*x^2 + sqrt(3)*(c^2)^(1/6)*x + 1)/((c^2)^(1/3)*c^2) - (sqrt(3)*sqrt(c^2)*c
^2*d^3*e^2 - (c^2)^(2/3)*c^2*d^4*e + sqrt(3)*(c^2)^(1/6)*c^2*d*e^4 - 2*(c^2
)^(1/3)*c^2*d^2*e^3 - c^2*e^5)*log((c^2)^(1/3)*x^2 - sqrt(3)*(c^2)^(1/6)*x
+ 1)/((c^2)^(1/3)*c^2) - (2*c^4*d^5 + sqrt(3)*(c^2)^(5/6)*c^2*d^4*e + (c^2)
^(2/3)*c^2*d^3*e^2 - sqrt(3)*(c^2)^(1/6)*c^2*e^5 - (3*(c^2)^(1/3)*c^2 - 2*(
c^2)^(4/3))*d*e^4)*log((2*(c^2)^(1/3)*x + sqrt(3)*(c^2)^(1/6) - sqrt(-(c^2)
^(1/3)))/(2*(c^2)^(1/3)*x + sqrt(3)*(c^2)^(1/6) + sqrt(-(c^2)^(1/3))))/((c^
2)^(1/3)*c^2*sqrt(-(c^2)^(1/3))) - (2*c^4*d^5 - sqrt(3)*(c^2)^(5/6)*c^2*d^4
```

```
*e + (c^2)^(2/3)*c^2*d^3*e^2 + sqrt(3)*(c^2)^(1/6)*c^2*e^5 - (3*(c^2)^(1/3)
*c^2 - 2*(c^2)^(4/3))*d*e^4)*log((2*(c^2)^(1/3)*x - sqrt(3)*(c^2)^(1/6) - s
qrt(-(c^2)^(1/3)))/(2*(c^2)^(1/3)*x - sqrt(3)*(c^2)^(1/6) + sqrt(-(c^2)^(1/
3))))/((c^2)^(1/3)*c^2*sqrt(-(c^2)^(1/3))))/(c^2*d^6*e + e^7))*c - 4*arctan
(c*x^3)/(e^2*x + d*e))*b - a/(e^2*x + d*e)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```
11
12 GradeAntiderivative := proc(result,optimal)
13 local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
       debug:=false;
14
15      leaf_count_result:=leafcount(result);
16      #do NOT call ExpnType() if leaf size is too large. Recursion problem
17      if leaf_count_result > 500000 then
18          return "B";
19      fi;
20
21      leaf_count_optimal:=leafcount(optimal);
22
23      ExpnType_result:=ExpnType(result);
24      ExpnType_optimal:=ExpnType(optimal);
25
26      if debug then
27         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
       ExpnType_optimal);
28      fi;
29
30 # If result and optimal are mathematical expressions,
31 #  GradeAntiderivative[result,optimal] returns
32 #   "F" if the result fails to integrate an expression that
33 #       is integrable
34 #   "C" if result involves higher level functions than necessary
35 #   "B" if result is more than twice the size of the optimal
36 #       antiderivative
37 #   "A" if result can be considered optimal
38
39   #This check below actually is not needed, since I only
40   #call this grading only for passed integrals. i.e. I check
41   #for "F" before calling this. But no harm of keeping it here.
42   #just in case.
43
44
45   if not type(result,freeof('int')) then
46      return "F";
47   end if;
48
49
50   if ExpnType_result<=ExpnType_optimal then
51       if debug then
52            print("ExpnType_result<=ExpnType_optimal");
53       fi;
54       if is_contains_complex(result) then
55          if is_contains_complex(optimal) then
```

```
56            if debug then
57               print("both result and optimal complex");
58            fi;
59            #both result and optimal complex
60            if leaf_count_result<=2*leaf_count_optimal then
61               return "A";
62            else
63               return "B";
64            end if
65         else #result contains complex but optimal is not
66            if debug then
67               print("result contains complex but optimal is not");
68            fi;
69            return "C";
70         end if
71       else # result do not contain complex
72            # this assumes optimal do not as well
73            if debug then
74                print("result do not contain complex, this assumes optimal do not
     as well");
75            fi;
76            if leaf_count_result<=2*leaf_count_optimal then
77               if debug then
78                  print("leaf_count_result<=2*leaf_count_optimal");
79               fi;
80               return "A";
81            else
82               if debug then
83                  print("leaf_count_result>2*leaf_count_optimal");
84               fi;
85               return "B";
86            end if
87       end if
88   else #ExpnType(result) > ExpnType(optimal)
89       if debug then
90          print("ExpnType(result) > ExpnType(optimal)");
91       fi;
92       return "C";
93   end if
94
95 end proc:
96
97 #
98 # is_contains_complex(result)
99 # takes expressions and returns true if it contains "I" else false
100 #
101 #Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
149   elif AppellFunctionQ(op(0,expn)) then
150     max(6,apply(max,map(ExpnType,[op(expn)])))
151   elif op(0,expn)='int' then
152     max(8,apply(max,map(ExpnType,[op(expn)]))) else
153   9
154   end if
155 end proc:
156
157
158 ElementaryFunctionQ := proc(func)
159   member(func,[
160         exp,log,ln,
161         sin,cos,tan,cot,sec,csc,
162         arcsin,arccos,arctan,arccot,arcsec,arccsc,
163         sinh,cosh,tanh,coth,sech,csch,
164         arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
165 end proc:
166
167 SpecialFunctionQ := proc(func)
168   member(func,[
169         erf,erfc,erfi,
170         FresnelS,FresnelC,
171         Ei,Ei,Li,Si,Ci,Shi,Chi,
172         GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
173         EllipticF,EllipticE,EllipticPi])
174 end proc:
175
176 HypergeometricFunctionQ := proc(func)
177   member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
178 end proc:
179
180 AppellFunctionQ := proc(func)
181   member(func,[AppellF1])
182 end proc:
183
184 # u is a sum or product.  rest(u) returns all but the
185 # first term or factor of u.
186 rest := proc(u) local v;
187   if nops(u)=2 then
188      op(2,u)
189   else
190      apply(op(0,u),op(2..nops(u),u))
191   end if
192 end proc:
193
194 #leafcount(u) returns the number of nodes in u.
195 #Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
42          if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
43             return True
44          else:
45             return False
46
47      except AttributeError as error:
48          return False
49
50  def expnType(expn):
51      debug=False
52      if debug:
53          print("expn=",expn,"type(expn)=",type(expn))
54
55      if is_atom(expn):
56         return 1
57      elif isinstance(expn,list):
58          return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
59      elif  is_sqrt(expn):
60          if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
61              return 1
62          else:
63              return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
64      elif isinstance(expn,Pow):   #type(expn,'`^`')
65          if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
66              return expnType(expn.args[0])   #ExpnType(op(1,expn))
67          elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
68              if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
69                  return 1
70              else:
71                  return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
    ))
72          else:
73              return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
    ExpnType(op(1,expn)),ExpnType(op(2,expn)))
74      elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
    (expn,'`*`')
75          m1 = expnType(expn.args[0])
76          m2 = expnType(list(expn.args[1:]))
77          return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
78      elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
79          return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
80      elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
81          m1 = max(map(expnType, list(expn.args)))
82          return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
83      elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
    expn))
84          m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
    Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
2  #              Albert Rich to use with Sagemath. This is used to
3  #              grade Fricas, Giac and Maxima results.
4  #Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
5  #              'arctan2','floor','abs','log_integral'
6
7  from sage.all import *
8  from sage.symbolic.operators import add_vararg, mul_vararg
9
10 def tree(expr):
11     debug=False;
12     if debug:
13          print ("Enter tree(expr), expr=",expr)
14          print ("expr.operator()=",expr.operator())
15          print ("expr.operands()=",expr.operands())
16          print ("map(tree, expr.operands()=",map(tree, expr.operands()))
17
18     if expr.operator() is None:
19         return expr
20     else:
21         return [expr.operator()]+list(map(tree, expr.operands()))
22
23 def leaf_count(anti):
24     debug=False;
25
26     if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))
27
28     if len(anti) == 0: #special check for optimal being 0 for some test cases.
29        if debug: print ("len(anti) == 0")
30        return 1
31     else:
32         if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
    flatten(tree(anti)))))
33         return round(1.35*len(flatten(tree(anti)))) #fudge factor
34                     #since this estimate of leaf count is bit lower than
35                     #what it should be compared to Mathematica's
36
37 def is_sqrt(expr):
38     debug=False;
39     if expr.operator() == operator.pow:   #isinstance(expr,Pow):
40         if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
41             if debug: print ("expr is sqrt")
42             return True
43         else:
44             return False
45     else:
46         return False
47
```

```
48 def is_elementary_function(func):
49     debug = False
50 
51     m = func.name() in ['exp','log','ln',
52             'sin','cos','tan','cot','sec','csc',
53             'arcsin','arccos','arctan','arccot','arcsec','arccsc',
54             'sinh','cosh','tanh','coth','sech','csch',
55             'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
56         'arctan2','floor','abs'
57         ]
58     if debug:
59         if m:
60             print ("func ", func , " is elementary_function")
61         else:
62             print ("func ", func , " is NOT elementary_function")
63 
64 
65     return m
66 
67 def is_special_function(func):
68     debug = False
69 
70     if debug: print ("type(func)=", type(func))
71 
72     m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
73            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
74            'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
75            'polylog','lambert_w','elliptic_f','elliptic_e',
76            'elliptic_pi','exp_integral_e','log_integral']
77 
78     if debug:
79         print ("m=",m)
80         if m:
81             print ("func ", func ," is special_function")
82         else:
83             print ("func ", func ," is NOT special_function")
84 
85 
86     return m
87 
88 
89 def is_hypergeometric_function(func):
90     return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']
91 
92 def is_appell_function(func):
```

```
93      return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath
94
95  def is_atom(expn):
96
97      #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
98      try:
99          if expn.parent() is SR:
100             return expn.operator() is None
101         if expn.parent() in (ZZ, QQ, AA, QQbar):
102             return expn in expn.parent() # Should always return True
103         if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
104             return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
105         return False
106
107     except AttributeError as error:
108         return False
109
110
111 def expnType(expn):
112     debug=False
113
114     if debug:
115         print (">>>>>Enter expnType, expn=", expn)
116         print (">>>>>is_atom(expn)=", is_atom(expn))
117
118     if is_atom(expn):
119         return 1
120     elif type(expn)==list:   #isinstance(expn,list):
121         return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
122     elif  is_sqrt(expn):
123         if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
124             return 1
125         else:
126             return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
127     elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
128         if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
129             return expnType(expn.operands()[0])   #expnType(expn.args[0])
130         elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
131             if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
132                 return 1
```

```
133             else:
134                 return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
135         else:
136             return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
137     elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
138         m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
139         m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
140         return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
141     elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.
    func)
142         return max(3,expnType(expn.operands()[0]))
143     elif is_special_function(expn.operator()): #is_special_function(expn.func)
144         m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
145         return max(4,m1)   #max(4,m1)
146     elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
147         m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
148         return max(5,m1)   #max(5,m1)
149     elif is_appell_function(expn.operator()):
150         m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
151         return max(6,m1)   #max(6,m1)
152     elif str(expn).find("Integral") != -1: #this will never happen, since it
153                 #is checked before calling the grading function that is passed.
154                 #but kept it here.
155         m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
156         return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
157     else:
158         return 9
159
160 #main function
161 def grade_antiderivative(result,optimal):
162     debug = False;
163
164     if debug: print ("Enter grade_antiderivative for sagemath")
165
166     leaf_count_result  = leaf_count(result)
167     leaf_count_optimal = leaf_count(optimal)
168
169     if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)
```

```
170
171
172     expnType_result  = expnType(result)
173     expnType_optimal = expnType(optimal)
174
175     if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
        expnType_optimal)
176
177     if expnType_result <= expnType_optimal:
178         if result.has(I):
179             if optimal.has(I): #both result and optimal complex
180                 if leaf_count_result <= 2*leaf_count_optimal:
181                     return "A"
182                 else:
183                     return "B"
184             else: #result contains complex but optimal is not
185                 return "C"
186         else: # result do not contain complex, this assumes optimal do not as
        well
187             if leaf_count_result <= 2*leaf_count_optimal:
188                 return "A"
189             else:
190                 return "B"
191     else:
192         return "C"
```